Consensus under a fuzzy context: Taxonomy, analysis framework AFRYCA and experimental case of study

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ABSTRACT

Consensus reaching processes play an increasingly important role in the resolution of group decision making problems: a solution acceptable to all the experts participating in a problem is necessary in many real-life contexts. A large number of consensus approaches have been proposed to support groups in such processes, each one with its own characteristics, such as the methods utilized for the fusion of information regarding the preferences of experts. Given this variety of existing approaches in the literature to support consensus reaching processes, this paper considers two main objectives. Firstly, we propose a taxonomy that provides an overview and categorization of some existing consensus models for group decision making problems defined in a fuzzy context, taking into account the main features of each model. Secondly, the paper presents AFRYCA, a simulation-based analysis framework for the resolution of group decision making problems by means of different consensus models. The framework is aimed at facilitating a study of the performance of each consensus model, as well as determining the most suitable model/s for the resolution of a specific problem. An experimental study is carried out to show the usefulness of the framework.

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1. Introduction

Decision making is a common process in daily life, characterized by the existence of several alternatives and the need to decide which one/s are the best or should be chosen as the solution to a problem. Group Decision Making (GDM) problems, in which several individuals or experts with different points of view take part in a decision problem with the aim of achieving a common solution, frequently occur in many organizations nowadays [1,2]. Although decision problems may take place in different environments (certainty, risk or uncertainty), most real-life GDM problems are often defined in uncertain environments. Due to the difficulty of dealing with uncertainty of a non-probabilistic nature, which is mainly caused by the imprecision and vagueness of information, experts must express their preferences over alternatives by means of information domains that allow them to deal with such uncertainty. To do so, fuzzy modeling and linguistic information has been utilized in such situations [3–5].

Traditionally, GDM problems have been solved by applying an alternative selection process [6], in which the preferences of each expert over the alternatives are gathered and the best alternative or subset of alternatives is chosen [7]. This resolution scheme does not take into account the existing level of agreement between experts, therefore some experts may not accept the decision made because they might consider that their individual preferences have not been taken into account sufficiently [8,9]. For this reason, Consensus Reaching Processes (CRPs) were introduced as an additional phase in the resolution of GDM problems [9]. In a CRP, experts discuss and modify their preferences, frequently coordinated by a human moderator, bringing their opinions closer to each other with the aim of increasing the level of agreement in the group.

Consensus has become a major research topic within the field of GDM. As a result, a large number of models and approaches to supporting CRPs have been proposed by several authors in the last few decades [10–17]. The earliest proposals of consensus approaches were developed with the objective of reaching a full degree of agreement in the group, i.e. unanimity [18], which is normally difficult to achieve in practice. Therefore, more flexible notions of consensus in which different partial degrees of agreement can be obtained, have since been proposed [2,19]. Consensus measures that are based on such flexible notions of agreement indicate

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how close experts' opinions are to unanimity. To do this, consensus degrees can be assessed in different ways, e.g. with numerical values in the unit interval [16,20,21], or linguistically [22–25].

A large number of consensus models have been proposed for dealing with GDM problems in fuzzy contexts, therefore they may present a high variety of features, such as: (i) the type of consensus measures utilized to determine the level of agreement, based on the fusion of information about experts' preferences [19,23,26], (ii) the use of different mechanisms to guide the discussion process [27], or (i) the type of preference structures (e.g. preference relations, preference orderings, utility vectors, etc. [28]) or information domains (e.g. numerical or linguistic information [22,29]) used by experts to express their preferences over alternatives, amongst others. Additionally, some models are focused on multiple criteria GDM problems (MCGDM) [29,30], in which information fusion approaches are often utilized to combine preferences evaluated according to several criteria, whilst other models have been defined to deal with a particular type of real-life decision problems [10,31].

Given this variety of existing consensus models, it would be desirable to have a clear characterization of them, with regard to the needs of each problem to be solved (type of preferences used by experts, necessity of giving the experts different importance weights, etc.), so that the most suitable models would be identified for solving such a problem. Moreover, some challenges are still present in the research topic of consensus, such as: (i) the large number of existing consensus models in the literature without a clear vision about which ones would be suitable for solving a specific type of GDM problem and (ii) the lack of a frame of reference for the practical study of consensus models, which makes the analysis of their main features, their advantages and weaknesses, as well as comparisons amongst them, more difficult. Such a comparison would be particularly useful for evaluating new proposals of consensus models, in order to determine their main contributions with respect to other existing ones.

As a result of a thorough literature review on consensus approaches in a fuzzy context, in this paper we tackle two objectives: (i) proposing a taxonomy of existing works and (ii) presenting an analytic framework called AFRYCA:

- We firstly present a taxonomy that provides an overview of a number of consensus models, with the main goal of providing a characterization of them, as well as pointing out the main characteristics of each proposal. The consensus models reviewed will be categorized into four groups, based on a double axis: (i) the use or not of feedback mechanisms to guide discussion, and (ii) the type of consensus measures applied (based on the method utilized for the fusion of information related to the preferences of the experts).
- Secondly, the paper introduces a prototype of simulation-based analysis framework called AFRYCA (A FRamework for the analYsis of Consensus Approaches). The framework has been developed to simulate the resolution of GDM problems by means of the different consensus models implemented in it. Therefore, its main purpose is to enable the analysis of the performance of each consensus model, as well as studying the results obtained by using different models for the resolution of a particular problem. AFRYCA has been implemented using Java and R technologies, and it incorporates several extendable modules and features, such as libraries that implement consensus models or patterns of expert behavior for its simulation, amongst others.

An experimental study is also presented to illustrate the usefulness of the analysis framework developed. For this, six consensus models of those reviewed in the taxonomy, have been implemented and used for the resolution of GDM problems.

The paper is organized as follows: in Section 2, some basic concepts regarding consensus in GDM are reviewed, together with some related works on consensus measures. Section 3 presents a taxonomy of consensus models. The analysis framework AFRYCA is presented in Section 4, followed by an experimental study that illustrates its usefulness in Section 5. Section 6 contains remarks on some of the lessons learnt and future directions in the use of AFRYCA. Finally, some conclusions are drawn in Section 7.

2. Background

In this section, we revise some basic concepts and approaches presented in the literature about GDM problems and consensus, in order to provide readers with a better understanding of the consensus models reviewed in the taxonomy presented in Section 3.

2.1. Group decision making problems

A GDM problem can be formally defined as a decision situation where [1]:

(i) There exists a group of m individuals or *experts*, $E = \{e_1, \ldots, e_m\}$, having each one their own knowledge and attitudes.
(ii) There is a decision problem consisting of n *alternatives* or possible solutions to the problem, $X = \{x_1, \ldots, x_n\}$.
(iii) The experts try to achieve a common solution.

In a GDM problem, each expert $e_i \in E, i \in \{1, \ldots, m\}$, expresses his/her preferences over alternatives in X, by means of a preference structure. One of the most common preference structures in GDM is the so-called preference relation [29]. A preference relation P_i associated to expert e_i can be represented, for X finite, as an $n \times n$ matrix as follows:

$$P_i = \begin{pmatrix} - & \cdots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \cdots & - \end{pmatrix}$$

where each *assessment*, p_i^{lk}, represents the degree to which the alternative x_l is better than $x_k, l, k \in \{1, \ldots, n\}, l \neq k$, according to e_i. Other preference structures that have been considered in some GDM approaches are utility vectors [32] and preference orderings [33,34], amongst others.

Some problems are characterized by the existence of several attributes or criteria, $C = \{c_1, \ldots, c_q\}$ (e.g. location, neighborhood and size, in a problem about buying a new house). In such situations, experts must assess alternatives according to each of these criteria, $c_y \in C$, i.e. a Multi-Criteria Group Decision Making (MCGDM) problem is defined [1].

GDM problems are often defined in environments of uncertainty, characterized by the existence of vague and imprecise information. Such situations are also known as GDM problems in fuzzy contexts or *fuzzy GDM problems* in the literature [3]. In order to deal with such uncertainty, experts may utilize different information domains to provide their preferences out of the existing alternatives, depending on their knowledge area or level of expertise in the problem. Some information domains frequently utilized in GDM problems under uncertainty are [35,36]:

- *Numerical* [37]: Assessments are represented as numerical values belonging to a specific scale, e.g. values in the [0,1] interval or values in Saaty's 1–9 multiplicative scale [38].

- *Interval-valued* [39]: Assessments are represented as intervals, $I([0, 1])$.
- *Linguistic* [40,41]: Assessments are represented as linguistic terms $s_u \in S, u \in \{0, \ldots, g\}$, being $S = \{s_0, \ldots, s_g\}$ a set of linguistic terms with granularity g.

The solution for a GDM problem can be derived by applying either a direct approach or an indirect approach [6]. In a *direct approach*, the solution is directly obtained from the individual preferences of experts, without constructing a social opinion first. In an *indirect approach*, however, a social opinion or *collective preference* (as it will be referred to in the rest of the paper) is determined a priori from individual opinions, and utilized to find a solution for the problem. Regardless of the approach considered, the classical alternative selection process for reaching a solution to GDM problems is composed of two phases [7], as shown in Fig. 1:

(i) *Aggregation phase*: the preferences of experts are combined, by using an aggregation operator.
(ii) *Exploitation phase*: This phase consists in obtaining an alternative or subset of alternatives as the solution to the problem, by means of a selection criterion.

2.2. Consensus in GDM: consensus measures and related works

The selection process for GDM problems described above does not guarantee the existence of agreement amongst experts before obtaining a solution to the problem. Therefore, it may be that such a solution is not accepted by some experts in the group, because they might consider that their individual opinions have not been taken into account sufficiently [8,9,42]. In many real-life GDM problems, obtaining a solution which is highly accepted by the whole group is crucial. In such cases, an additional phase called the consensus phase must be introduced into the resolution process for GDM problems [9]. This phase usually consists of a process of discussion and modification of preferences by experts, with the aim of reaching a high level of collective agreement (further detail regarding this process will be given in Section 2.3).

The concept of consensus has been interpreted from different points of view, from total agreement (unanimity), which is usually difficult to achieve in practice, to more flexible interpretations. In [9], Saint et al. defined consensus as "a state of mutual agreement among members of a group, where all legitimate concerns of individuals have been addressed to the satisfaction of the group". Kacprzyk et al. introduced the notion of *soft consensus*, based on the concept of fuzzy majority [2], which states that consensus exists when "most of the important individuals agree as to (their testimonies concerning) almost all of the relevant options" [19].

Flexible notions of consensus imply that it can be measured as different levels of partial agreement in the group, which indicate how far the opinions of experts are from unanimity. Therefore, the definition of appropriate *consensus measures*, which compute the current level of agreement in the group from the individual preferences of experts, has been an important subject of research within the field of consensus in GDM. A large number of consensus measures ha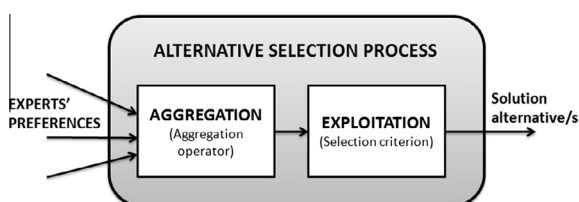ve been proposed by different authors in the literature [19,24,43–45]. Based on a literature review of different consensus measures proposed by several authors, we have classified them into two categories, depending on the type of computations and information fusion procedures applied to measure consensus:

1. *Consensus measures based on distances to the collective preference*: A collective preference, denoted as P_c, that represents the global opinion of the group is computed by aggregating all individual preferences of experts, P_i, i.e. $P_c = \phi\{P_1, \ldots, P_m\}$, with ϕ being an aggregation operator. Consensus degrees are then obtained by computing the distances between each individual preference and the collective preference, $d(P_i, P_c)$ [24,43,44].
2. *Consensus measures based on distances between experts*: For each different pair of experts in the group, $(e_i, e_j), i < j$, the degrees of similarity between their opinions are computed, based on distance metrics. Similarity values $L(P_i, P_j)$ are then aggregated to obtain consensus degrees [19,22,25,45].

Fig. 2 shows a general scheme of the computations carried out in both types of consensus measures described above. In the following subsections, some consensus measures belonging to each of these two categories are briefly reviewed.

2.2.1. Consensus measures based on distances to the collective preference

Spillman et al. proposed in [43] one of the earliest consensus measures based on mathematical procedures taken from fuzzy set theory [4], thus complying with a notion of consensus which is more flexible and realistic in practice than the idea of consensus as unanimous agreement, as considered in other earlier works [18]. In their proposal, Spillman et al. measure the degree of consensus for each expert separately, as the distance between his/her reciprocal fuzzy preference relation and an "ideal" consensus matrix with maximum consensus degree, determined a priori by means of matrix calculus. Another complementary measure is the fuzziness degree, whose value is larger if the consensus degree is lower and vice versa, which is also introduced and utilized as a criterion to quantify the level of group agreement.

One of the first consensus measures for linguistic preferences was presented by Herrera et al. in [24], assuming that experts might sometimes have a vague knowledge about the problem and they would prefer to use linguistic assessments instead of numerical ones. Alternatives and experts have fuzzy importance degrees, inspired by Kacprzyk's *soft consensus* approach [2,19] (which will be revised in Section 2.2.2). Two different consensus measures are calculated: *consensus degrees*, which indicate the current level of agreement; and *linguistic distances*, used to evaluate the distance from each expert's linguistic preference relation to the collective opinion. Both measures are assessed linguistically, by means of linguistic terms s_u belonging to a finite term set $S = \{s_0, \ldots, s_g\}$ defined a priori, and they are calculated at three levels (using the LOWA operator [46] to aggregate information) by applying three steps sequentially: (i) a counting process, (ii) a coincidence process and (iii) a computing process [24].

In [23], Herrera et al. extended the consensus measures described above, by incorporating a process to control the consistency of preferences. The consistency control process is carried out before measuring consensus.

Ben-Arieh et al. studied in [47] the problem of aggregating linguistic preferences, expressed as fuzzy sets in a common linguistic term set by a group of experts who have associated linguistic importance weights. Firstly, they extended the Fuzzy-LOWA operator [44] to consider such importance weights in the aggregation of individual preferences into a collective preference. Then, they defined a consensus measure in which individual preference order-
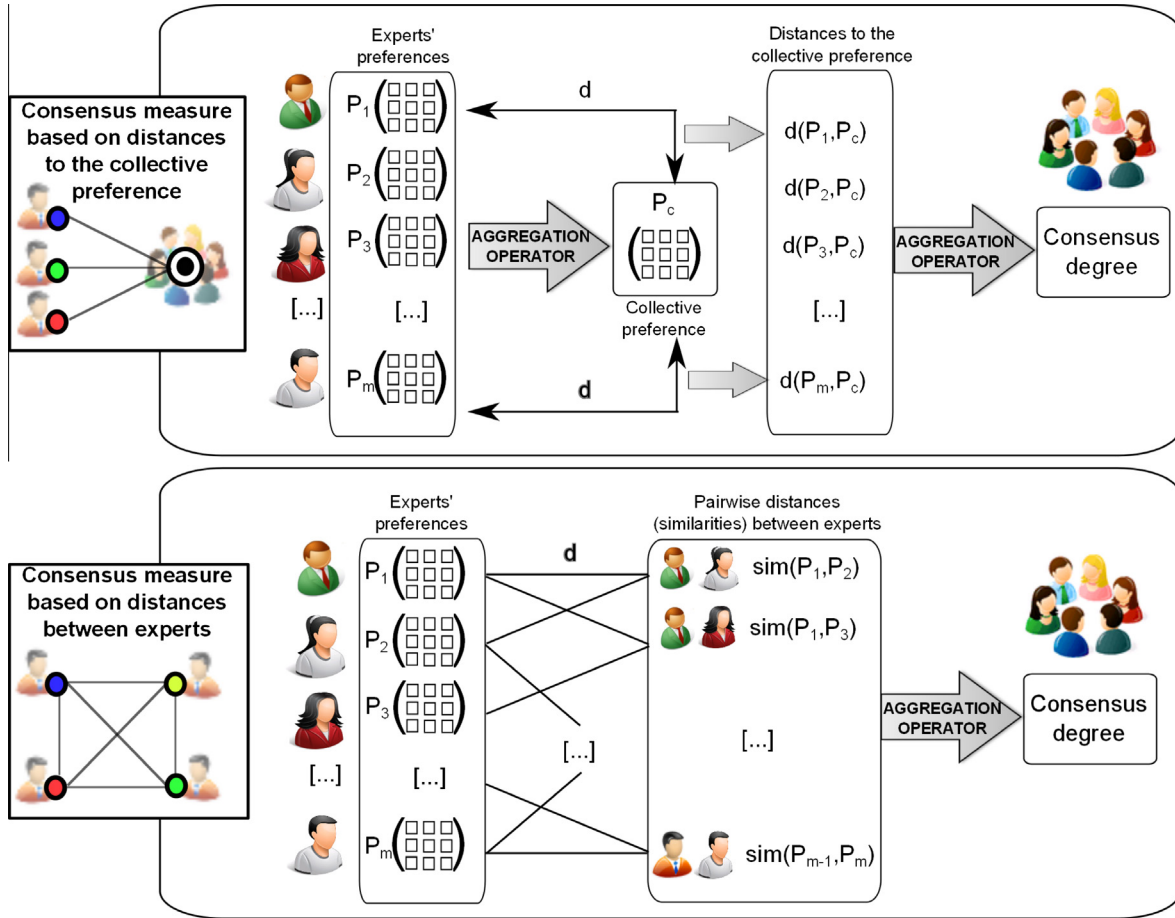

Fig. 1. Selection process for the resolution of GDM problems.

Fig. 2. Types of consensus measures.

ings and a collective preference ordering are compared. Such preference orderings are derived from their corresponding linguistic preferences. The degree of consensus C_l on an alternative x_l is computed as follows:

$$C_l = \sum_{i=1}^{m} \left[\left(1 - \frac{|O_i^l - O_c^l|}{n-1} \right) \times w_i \right] \tag{1}$$

with O_i^l and O_c^l being the ordered position of x_l, for expert e_i and the collective opinion respectively, and w_i the importance weight of e_i. The arithmetic mean operator is then used to compute the global consensus degree from all $C_l, l \in \{1 \ldots n\}$.

2.2.2. Consensus measures based on distances between experts

Kacprzyk et al. conducted extensive research into human-consistent measures of consensus that reflect the human perception of consensus in practice in a better way than consensus as unanimous agreement. As a result, they proposed the notion of *soft consensus*, based on the concept of fuzzy majority [2]. One of the first consensus measures for fuzzy preference relations based on this notion was formalized in [19]. The consensus degree is hierarchically computed at multiple levels, starting by α-degrees of sufficient agreement (with $\alpha \in [0,1]$) between two experts (e_i, e_j) on a single assessment p_i^{lk}:

$$sim_{ij}^{lk} = \begin{cases} 1 & \text{if } |p_i^{lk} - p_j^{lk}| \leqslant 1 - \alpha \leqslant 1, \\ 0 & \text{otherwise}. \end{cases} \tag{2}$$

The concept of fuzzy majority is reflected in the consensus measures by applying a fuzzy logic-based calculus of linguistically

quantified propositions [2,48], taking into account the fuzzy importance weights assigned to experts and alternatives. The computation scheme of this "soft" consensus measure was slightly simplified in [45].

A different approach from *soft consensus* was taken into account by Szmidt and Kacprzyk in [49], where they extended the measures for fuzzy preference relations defined by Spillman et al. [43], and developed a consensus measure for reciprocal intuitionistic fuzzy preference relations. Consensus is computed as a scalar value in $[0, 1]$, obtained from a consensus matrix of dimensions $m \times m$, in which each element cm_{ij} represents the degree of agreement between two experts e_i and e_j.

Herrera et al. proposed in [25] some consensus measures for linguistic GDM (linguistic consensus degrees and linguistic proximities, each one at three levels [24]), which pivot on determining degrees of fuzzy coincidence between pairs of experts, by means of a closeness measure between linguistic assessments. Different linguistic term sets can be used for the diverse elements of the GDM problem that are assessed linguistically, e.g. preferences, importance degrees of experts and alternatives, and consensus measures.

Another linguistic consensus measure was presented by Bordogna et al. in [22], being oriented towards MCGDM with linguistic preference matrices. This approach follows the concept of fuzzy majority, and it utilizes OWA operators [50] to aggregate preferences belonging to the different criteria. Such criteria are assessed linguistically by each expert. A linguistic consensus degree is computed for each alternative separately, based on degrees of agreement between pairs of experts.

Korshid et al. [51] presented a consensus measure based on coincidence between the positive and negative ideal degrees of

agreement. Experts use linguistic terms to express their preferences by means of a vector of linguistic assessments. Such assessments are associated to triangular fuzzy numbers, and interval judgements are obtained by applying the α-cut operator [4] on fuzzy numbers, thus constructing an $m \times n$ fuzzy judgement matrix from the interval-valued assessments of all experts. Positive and negative agreement matrices are constructed taking into account similarities between pairs of experts, and then the relative closeness degrees to these two matrices are computed for each alternative.

Chen et al. defined in [52] a consensus measure for GDM problems with uncertain linguistic preference relations, with assessments given by uncertain linguistic terms expressed as $p_i^{lk} = [s_u, s_v], s_u, s_v \in S, u \leqslant v$ [53]. They determine the similarity between two experts' assessments upon a deviation measure, $d(p_i^{lk}, p_j^{lk})$, and an overlapping measure, $o(p_i^{lk}, p_j^{lk})$, as follows:

$$sim_{ij}^{lk} = \gamma(1 - d(p_i^{lk}, p_j^{lk})) + (1 - \gamma)o(p_i^{lk}, p_j^{lk}) \qquad (3)$$

with $\gamma \in [0, 1]$ being the importance given to the deviation measure with respect to the overlapping measure, in the computation of similarity values. Consensus and proximity degrees are then computed at three levels. The Uncertain LOWA operator is utilized to aggregate uncertain linguistic preferences into a collective preference, which is necessary in order to calculate proximity degrees.

2.3. Consensus in GDM: Consensus Reaching Processes (CRPs)

As previously stated, reaching consensus normally implies that experts must modify their initial opinions over the course of a discussion process (i.e a CRP), bringing their positions closer to each other, towards a final collective opinion which satisfies the whole group [8,9,54].

Before initiating a CRP, it is important that some a priori assumptions are understood and accepted by the whole decision group [42]:

- Every member of the group *must* understand the process used to achieve an agreement, clarifying any possible doubts or questions before initiating it.
- Conducting a CRP implies that all experts *accept* the search for a common agreed solution, by means of collaboration.
- Experts *should move* from their initial positions, in order to make their preferences closer to each other.

A large number of consensus models have been proposed during recent decades [10–13,39,15–17]. Consensus models provide groups with the necessary guidelines to support them in CRPs carried out in different GDM frameworks.

The process to reach consensus is iterative and dynamic. Such a process is often coordinated by a human figure known as *moderator*, who is responsible for supervising and guiding the discussion between experts [42]. A general CRP scheme followed by all consensus models revised in the taxonomy (see Section 3), is shown in Fig. 3. Its main phases are described below:
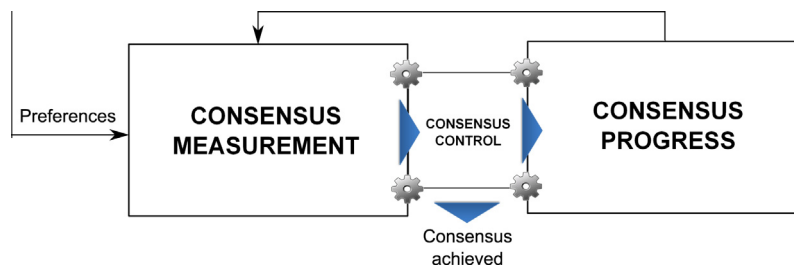
1. *Consensus Measurement*: Preferences of all experts, $P_i, i \in \{1, \ldots, m\}$, are gathered to compute the current level of agreement in the group, by using consensus measures (see Section 2.2).
2. *Consensus Control*: The consensus degree is compared with a threshold level of agreement μ, defined a priori. If the level of consensus desired has been achieved, the group moves onto the selection process; otherwise, it is necessary to carry out another round of discussion. In order to prevent an excessive number of discussion rounds, a parameter indicating the maximum number of rounds allowed, *Maxround* $\in \mathbb{N}$, can also be taken into account.
3. *Consensus Progress*: A procedure is applied in order to increase the level of agreement in the following round of the CRP. Traditionally, such a procedure has consisted of applying a *feedback generation* process, in which the moderator identifies the assessments of experts which are farthest from consensus and advises them to modify such assessments [9,42]. Many existing consensus models incorporate feedback mechanisms based on this process [28,27,32,55]. However, some other proposed models do not incorporate such mechanisms, and instead they implement approaches that update information (e.g. assessments of experts) to increase consensus in the group automatically [44,56,57].

3. A taxonomy of consensus approaches in a fuzzy context

In this section, we propose a taxonomy that reviews different consensus models proposed by a variety of authors to support CRPs in GDM problems defined in a fuzzy environment. The main goal of the taxonomy is to categorize such models, so that those with similar characteristics are grouped in the same category.

Fig. 4 shows the structure of the taxonomy. In order to categorize the consensus models reviewed, we have considered two different kinds of criteria for constructing the taxonomy:

- *Feedback versus No Feedback*: Many consensus models define a feedback mechanism to support experts in the discussion and modification of their opinions. Such feedback mechanisms generate and provide experts with some advice, indicating to them how to modify their preferences in order to bring them closer to consensus, hence they must *supervise* this advice and decide whether to apply it or not [27,28,32,55]. Some other consensus models do not consider the use of feedback mechanisms, but instead implement other types of mechanisms that *automatically* update the preferences and/or importance weights of those experts whose opinions are not close enough to the rest of the group, thus making the human intervention of experts unnecessary in these models [44,56,57].
- *Type of consensus measure*: A key element in all consensus models is the consensus measure utilized to compute the level of agreement in the group. As previously reviewed in Section 2.2, such measures are normally either based on computing distances to the collective preference (see Section 2.2.1) or based on computing distances between experts (see Section 2.2.2).
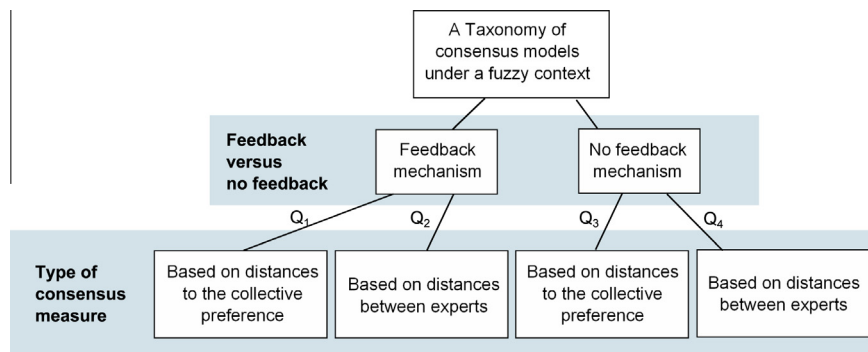

Fig. 3. General CRP scheme.

Fig. 4. A taxonomy of approaches for consensus reaching.

Table 1
Overview of consensus models reviewed in the taxonomy.

	Consensus measure based on distances to the collective preference	Consensus measure based on distances between experts
Feedback mechanism	(Q_1) Bryson [32,58] Herrera-Viedma et al. [28] Choudhury et al. [31] Dong et al. [62] Parreiras et al. [12,29] Jiang et al. [64]	(Q_2) Carlsson et al. [55] Eklund et al. [10,59] Herrera-Viedma et al. [60,61] Chiclana et al. [63] Mata et al. [27] Cabrerizo et al. [65] Pérez et al. [66] Alonso et al. [67] Kacprzyk et al. [13,68,69] Fu et al. [39,30,70,14]
No feedback mechanism	(Q_3) Lee [71] Ben-Arieh et al. [44] Chen et al. [74] Xia et al. [76], Xu et al. [77,78] Dong et al. [79], Zhang et al. [56] Gong et al. [15], Xu et al. [21] Wu and Xu [11,20,57,80,81]	(Q_4) Chen et al. [72] Zhang et al. [73] Palomares et al. [16,75]

Taking into account the two criteria described above, the classification of consensus models in the taxonomy is based on two axes, so that they are combined into four different quadrants that will categorize the consensus models revised in this paper (see Table 1):

- Q_1: Consensus models with feedback mechanism and a consensus measure based on computing distances to the collective preference, reviewed in Section 3.1.
- Q_2: Consensus models with feedback mechanism and a consensus measure based on computing pairwise similarities, reviewed in Section 3.2.
- Q_3: Consensus models without a feedback mechanism and with a consensus measure based on computing distances to the collective preference, reviewed in Section 3.3.
- Q_4: Consensus models without a feedback mechanism and with a consensus measure based on computing pairwise similarities, reviewed in Section 3.4.

Remark 1. For several consensus models reviewed throughout the following subsections, some figures with detailed schemes of their phases will be shown. The reason for showing the structure of these specific models in further detail rather than the other ones, is that they are already implemented in the initial version of the simulation-based analysis framework AFRYCA (see Section 4), and they will be utilized in the case study conducted in Section 5.

3.1. Q_1: feedback mechanism and consensus measure based on distances to the collective preference

In this section, we briefly review an assortment of consensus models characterized by: (i) the use of a feedback mechanism that provides some guidelines for experts on bringing their preferences closer to the rest of the group, and (ii) consensus measures based on the computation of distances between each expert's preference and the collective preference (see Fig. 5).

Bryson [32] proposed a model to assess the degree of group consensus and support group discussions under the Analytic Hierarchy Process (AHP) framework [38]. The model gathers, for each expert $e_i \in E$ a normalized numerical preference vector. Individual vectors are aggregated into a collective preference vector. Two thresholds and three consensus indicators are defined to decide whether the degree of consensus is sufficient or not, based on similarities between each individual vector and the collective vector. Bryson stated that the consensus preference vector should reflect an agreement that results from human interaction [32], hence the need for carrying out a negotiation process guided by a moderator [9], encouraging experts to interact with each other. Further guidelines and strategies to support such a negotiation (such as cooperation, communication and so on) by means of decision support tools in different scenarios, were later proposed by Bryson in [58], in which the use of qualitative assessments by experts, associated to numerical ranges (e.g. Poor: [0,40], Good: [60,80], etc.), was also introduced.
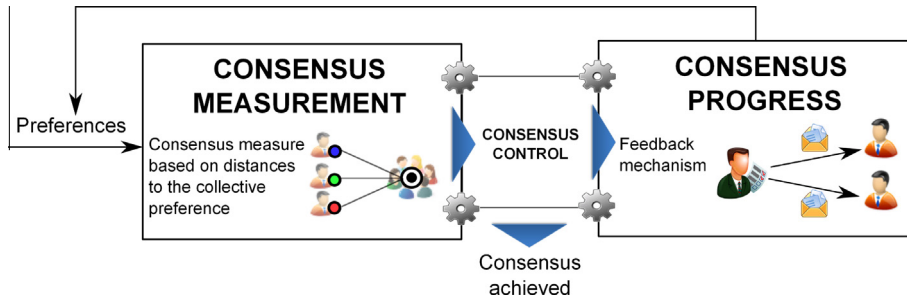
Fig. 5. General scheme of consensus models in Q_1.

The consensus model proposed by Herrera-Viedma et al. in [28] (represented in Fig. 6) allows experts to express their preferences by using different preference structures: (i) preference orderings O_i, (ii) utility functions U_i, (iii) fuzzy preference relations P_i and (iv) multiplicative preference relations A_i. Each expert chooses his/her most suitable preference structure according to the level of expertise he/she has in the problem. All preferences are conducted into fuzzy preference relations by means of several transformation functions. Furthermore, preference orderings of alternatives are obtained from individual fuzzy preference relations by computing the quantifier-guided dominance and non-dominance degrees for each alternative x_l (denoted in Fig. 6 as $QGDD_l$ and $QGNDD_l$, respectively). Such preference orderings are compared with a collective preference ordering to compute the consensus degrees. The model also introduces a feedback mechanism, based on proximity measures and a set of directions rules to suggest to experts how to increase/decrease some of their assessments.

Inspired by the consensus model with different preference structures proposed in [28], and considering its consensus measures, Choudhury et al. [31] proposed a consensus support system aimed at solving MCGDM problems in the context of advanced technology selection. Its main novelties with respect to previous models include the use of a multi-agent architecture [82] in which software agents with specific roles implement the different phases of the consensus model, as well as the aggregation of proximity degrees between experts and the collective preference, by means of the neat OWA operator, to obtain consensus degrees [83].

Dong et al. presented in [62] two consensus models for AHP-GDM with multiplicative preference relations [38]. The difference between the models is the nature of the consensus measure, which can be either ordinal or cardinal. Furthermore, unlike the above reviewed proposals, consensus measures are characterized by the application of a prioritization method that derives a prioritization vector of alternatives (instead of a preference ordering) from each preference relation. The collective preference is computed by
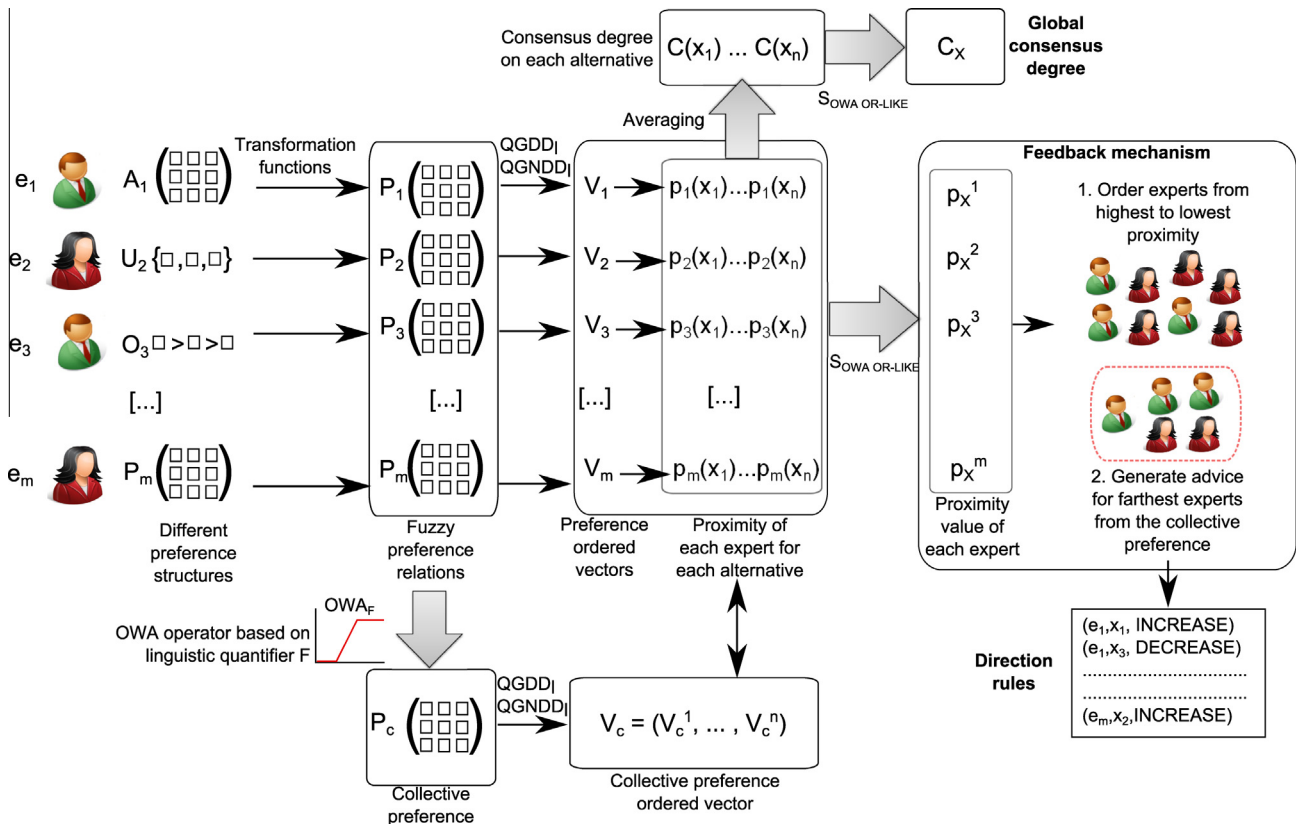

Fig. 6. Computation of consensus degrees and feedback mechanism in the model of Herrera-Viedma et al. [28].

means of the Weighted Geometric Mean operator. The proposed feedback mechanism identifies the expert farthest from consensus, determines some updated values for his/her preferences and shows the updated values to the human expert, who decides whether to accept or not the recommended changes.

Parreiras et al. proposed two consensus models for MCGDM problems. In their first model [12], experts utilize preference matrices with linguistic multi-granular assessments for each alternative and criterion, with the semantics of the linguistic terms given by trapezoidal membership functions. Since experts have importance weights according to their influence or position in the group, the authors suggested two methods to obtain them: either based on a discordance measure or by means of an optimization algorithm. The model presented in [29] introduces a measure of comparability to identify experts who experience difficulties in expressing their preferences (which are given by nonreciprocal fuzzy preference relations). In order to deal with such experts, other group members who are more sure of their opinion are invited to assist them. In both works, when the degree of consensus is insufficient, the moderator analyzes the concordance index of each expert with the collective preference, and suggests that the most discordant expert modifies his/her assessments.

More recently, Jiang et al. [64] defined a compatibility measure between intuitionistic multiplicative preference relations, and proposed two consensus models in which consensus degrees are measured for each expert separately, based on this compatibility measure. As occurred with [12,29,62], these models detect the farthest expert from the group opinion and invite him/her to modify his/her assessments. The second consensus model presented in [64] introduces identification rules in the feedback mechanism, in order to identify multiple discordant experts at the same discussion round and make the CRP more efficient.

3.2. Q_2: feedback mechanism and consensus measure based on distances between experts

A large number of consensus models in the literature calculate the closeness between all the different pairs of experts in the group for the measurement of consensus [19,22,23,60,84]. This section revises some consensus models that present this type of consensus measure and incorporate a feedback mechanism to guide experts across the CRP (Fig. 7).

Carlsson et al. [55] developed one of the first distributed consensus support systems to assist a group of experts connected to a local computer network. Its underlying consensus model follows an AHP framework for MCGDM problems in which experts provide preference matrices with assessments for each alternative and criterion, as well as the subjective importance weights they want to consider for each criterion. The consensus degree in the group is given by the maximum pairwise geometric distance between

experts, i.e. $\max_{ij} d(P_i, P_j)$. The feedback mechanism finds the farthest expert from consensus and suggests to him/her how to bring his/her preferences towards a central point between the rest of the experts' preferences. Based on the consensus measure defined by Carlsson et al. in which consensus is given by the maximum distance between two experts, Eklund et al. developed some models for consensus reaching in committees [59] and dynamic political contexts with coalition formation [10]. Their works include a detailed comparison between their consensus model and several voting schemes and rules, e.g. majority vote, plurality vote and Borda rule [85].

Herrera-Viedma et al. presented the first model aimed at letting experts with diverse levels of expertise express their preferences by means of different linguistic term sets (multi-granular linguistic preference relations) [60]. In order to deal with multi-granular linguistic information, they introduced a unification phase to conduct preferences into fuzzy sets in a common linguistic term set. This consensus model adopted some features which have been later considered by the authors in several works, such as: (i) a scheme for the computation of consensus degree at three levels (assessment, alternative and preference relation) upon pairwise similarities of experts, and (ii) a feedback mechanism consisting of identification and direction rules for experts, based on the computation of proximity degrees with the collective preference.

Several works have since been proposed, based on the consensus measure and feedback mechanism defined in Herrera-Viedma et al.'s model [60]. Their work in [61] is characterized by dealing with incomplete fuzzy preference relations whose missing assessments are computed by applying an estimation procedure. The model of Chiclana et al. [63] (see Fig. 8), incorporates a consistency control process applied before beginning the CRP to ensure consistency in individual fuzzy preference relations, and proposes an adaptive feedback mechanism in which the direction rules generated for experts depend on the level of agreement achieved at each round, which is compared with several consensus thresholds, $\theta_1 < \theta_2 < \mu$. The adaptive consensus model proposed by Mata et al. [27] considers the use of multi-granular linguistic information [61], and implements the adaptive feedback mechanism proposed in [63]. The consensus model of Cabrerizo et al. [65] is capable of dealing with unbalanced fuzzy linguistic information, given by linguistic terms distributed in a non-symmetrical and non-uniform way around a central term. Computational processes on unbalanced linguistic information are carried out by means of the 2-tuple linguistic model [86,87]. A mobile consensus support system model for dynamic GDM, was presented by Pérez et al. in [66]. The system allows experts connected to their own mobile device to use different preference structures to provide their opinions [28], and it considers dynamic problems in which the set of alternatives X may vary over time. Finally Alonso et al. proposed in [67] a linguistic consensus model for Web 2.0 communities, in which the set of experts might vary during the CRP. A delegation scheme
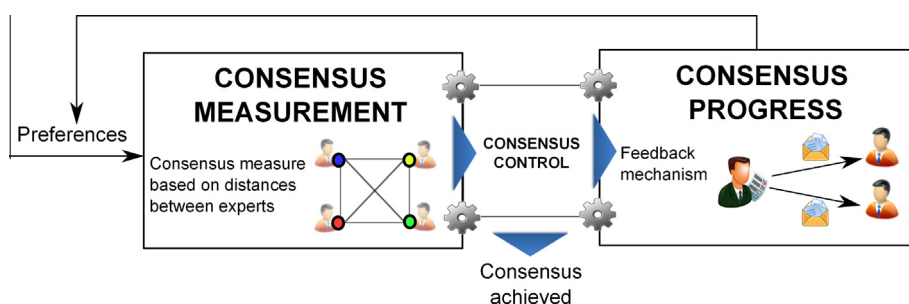

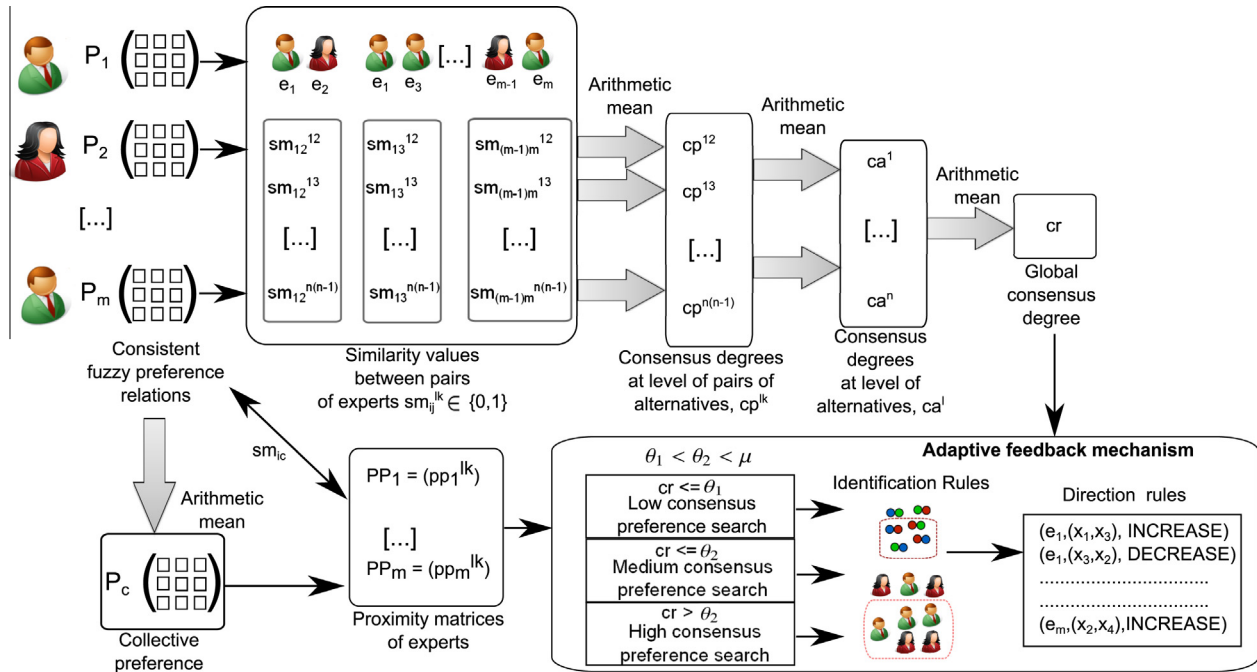**Fig. 7.** General scheme of consensus models in Q_2.

Fig. 8. Adaptive consensus model of Chiclana et al. [63].

based on trust weights between similar experts is defined to simplify GDM processes with large groups.

Kacprzyk et al. developed several consensus models based on their notion of *soft consensus* and fuzzy majority (see Section 2.2.2). In [68], they proposed a consensus model in which the moderator identifies experts and alternatives with difficulties in achieving a consensus by means of linguistic data summaries [88]. This proposal does not assign importance weights to experts and alternatives. Instead, two linguistic quantifiers F_1 and F_2 are utilized to capture the concept of fuzzy majority in the computation of consensus degrees at multiple levels [68], as illustrated in Fig. 9. The

authors also proposed some models of consensus support systems that implement their previous ideas. For instance, in [13,69] a concept of Web-based consensus support system that not only implements previous models, but also includes a guidance system based on several approaches, such as rule generation and collaborative filtering, is shown. In [13], ontologies are utilized to formalize knowledge managed by the system with regard to the consensus reaching processes and each particular GDM problem. In addition, the system incorporates a feedback mechanism consisting of computing quantifier-guided degrees of agreement over pairs of alternatives, identifying the pairs of alternatives in which the experts
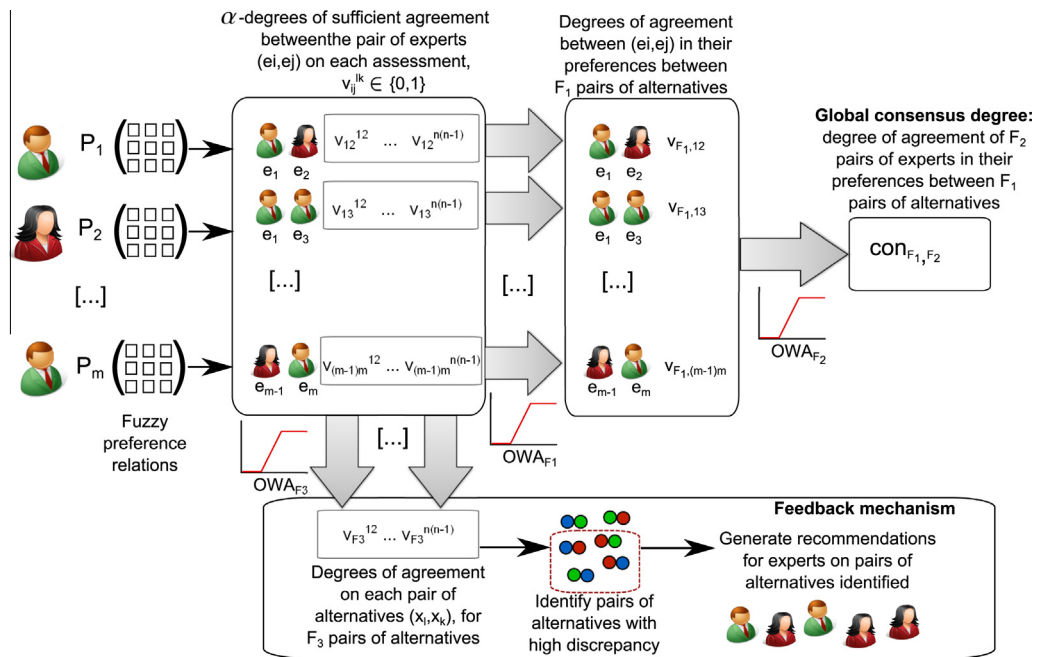

Fig. 9. Computation of consensus degree based on the concept of fuzzy majority, and feedback mechanism proposed by Kacprzyk et al. in [68,13], respectively.

present a higher degree of discrepancy, and providing recommendations to experts, based on several rules (see feedback mechanism in Fig. 9).

In [39,30,70,14], Fu et al. developed four consensus models for MCGDM problems in evidential reasoning contexts, where assessments of alternatives according to different criteria are given by distributed vectors of belief degrees, based on Dempster–Shafer evidence theory [89]. Such belief degrees can be either numerical [30,70] or interval-valued [39]. Assessments of pairs of experts are compared by means of a compatibility measure. Consensus degrees are then computed at three levels, similarly to [60]. In [39,70], they introduce a feedback mechanism consisting of identification rules and direction rules for experts, taking into account assessments related to criteria with the highest importance weights only. In [14], they extend the feedback mechanism, so that if consensus is not reached after some consecutive rounds of generating feedback, weights of experts are adjusted based on an optimization algorithm to ensure convergence to consensus.

3.3. Q_3: no feedback mechanism and consensus measure based on distances to the collective preference

Some consensus models do not incorporate a feedback mechanism and are designed to carry out the whole CRP automatically, so that the preferences and/or importance weights of experts are adjusted in order to reach a high level of agreement without the need for human intervention. This section revises several consensus models characterized by: (i) not incorporating any feedback mechanism and (ii) defining consensus measures based on the computation of distances to the collective preference (see Fig. 10).

In [71], Lee developed an iterative algorithmic approach to finding an optimal level of group consensus by adjusting the importance weights of experts and computing a collective preference based on them, so that the weighted sum of distances to the collective preference becomes minimal. The collective preference is given by the weighted average of individual preferences, which are expressed as trapezoidal fuzzy numbers. The consensus reaching algorithm is applied for each alternative separately.

Ben-Arieh et al. presented a consensus model for autocratic GDM [44] in a linguistic framework. Experts use linguistic preference relations, from which preference orderings are obtained to compute distances to the collective preference. Then consensus degrees are computed at the alternative and global level. If consensus is not enough, the degree of contribution of each expert towards consensus is determined, and weights of the least cooperating experts are penalized. More recently, Chen et al. defined in [74] an aggregation operator called ILLOWA (Interval Linguistic Labels Ordered Weighted Averaging) to facilitate the management of preferences expressed as interval linguistic labels, together with a consensus model that extends the one presented in [44] to manage this type of information.

Xu [77] considered the problem of consensus reaching in MCGDM, and developed a model that automatically updates all experts' assessments at the end of each consensus round if the level of agreement is not sufficient. To do so, an update coefficient $\eta \in (0, 1)$, which partially takes into account values of the collective preference to update experts' assessments, is defined and utilized. A convergent iterative algorithm that automates the whole CRP is proposed. Unlike previous automatic consensus approaches, the importance weights of the experts remain fixed across the CRP. They are utilized to compute the collective preference. Consensus is only achieved when *all* distances between experts and the collective preference fall below a threshold, i.e. $d(P_i, P_c) \leqslant \mu, \forall e_i \in E$. An extension of this work was proposed by Xia et al. in [76], in which an automatic consistency improvement algorithm on reciprocal fuzzy preference relations is also defined.

The work of Xu et al. [78] proposes a number of goal and quadratic programming models oriented towards the maximization of consensus in groups of experts whose preferences are given in the form of fuzzy and multiplicative preference relations. Such programming models aim to find the optimal weights of experts that minimize their deviation with respect to the collective preference.

Wu and Xu have proposed several automatic consensus models in the last few years [11,20,57,80,81], in which the process used to compute and control individual consensus degrees similar to [77] in all of them. The model in [80] is aimed at the resolution of MCGDM problems with cost/benefit criteria, hence a normalization of assessments in the unit interval is applied before proceeding to measure consensus. Its mechanism to bring preferences closer to each other consists of obtaining at each CRP round a weighted distance matrix *DM*. Then its maximum element is identified, and the corresponding assessment is updated by assigning the value of the collective assessment to the preferences of those experts with the largest distance from the group preference. Their subsequent works [11,20,57,81] utilize a simpler mechanism that updates the preferences of all experts whose distance to consensus exceeds a specified threshold. The updating of assessments is based on the updating coefficient, η [77]. Each of these proposals is characterized by the use of a different preference structure: linguistic preference relations [11], multiplicative preference relations [57], uncertain linguistic preference relations [81], and reciprocal fuzzy preference relations [20]. Fig. 11 shows the procedure used to compute consensus degrees and update preferences, corresponding to the consensus model based on reciprocal fuzzy preference relations.

The work of Dong et al. [79] focuses on the use of two different representational models to deal with linguistic preferences (continuous linguistic model [90] and 2-tuple fuzzy linguistic model [86,87]). They define a consensus measure based on an aggregation operator called Extended-OWA, to obtain the collective preference from continuous linguistic information. As stated in [77], all the experts must be close enough to the collective preference in order to reach a consensus, otherwise a quadratic programming algorithm
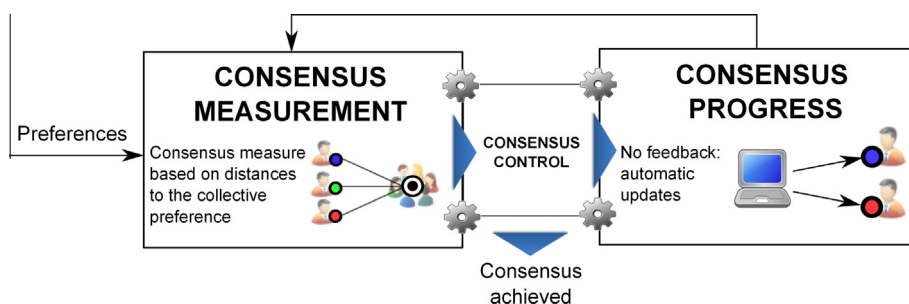

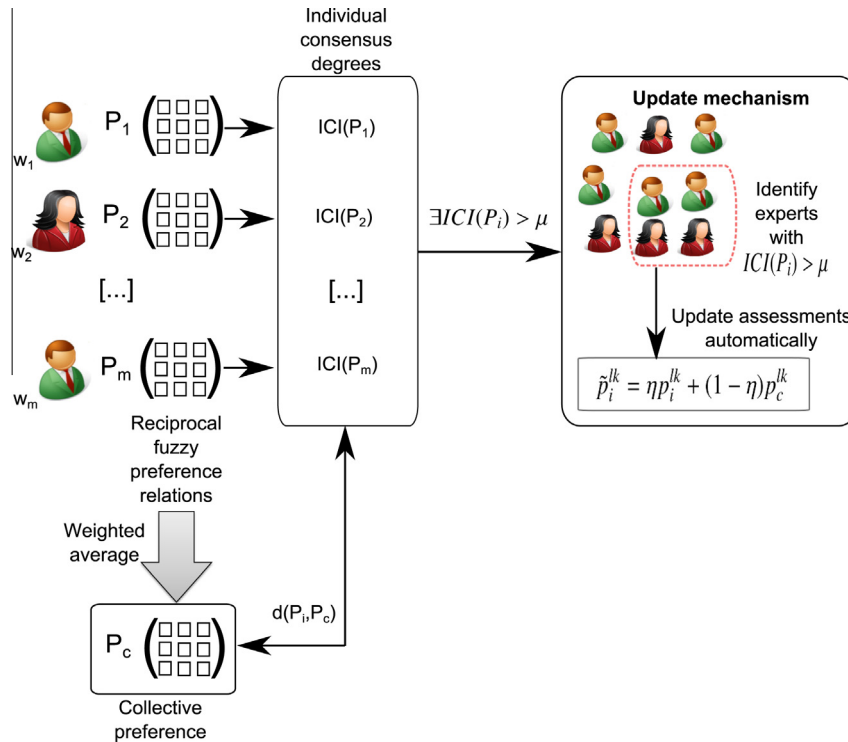**Fig. 10.** General scheme of consensus models in Q_3.

Fig. 11. Consensus model of Wu et al. [20].

that seeks the minimum required changes to individual preferences to find an agreement, is applied. Such an algorithm has since been considered by Zhang et al. in [56], in which a more generic consensus model under numerical preferences and the use of OWA operators is proposed.

Gong et al. formulated in [15] an optimization algorithm that, given a set of experts with associated weights and preferences expressed as 2-tuple linguistic preference relations, minimizes the deviation between all individual preferences and the collective preference. The optimization technique is applied to the values of experts' weights only, and no consensus thresholds are defined to decide on the existence of sufficient agreement, therefore the process ends when optimal weights are found. The additive consistency of preferences is also controlled.

The work of Xu et al. in [21] (see Fig. 12) proposes two distance-based consensus models for fuzzy and multiplicative preference relations, respectively. Two consensus measures are used in both models: Individual Consensus Indices $ICI(P_i) = d(P_i, P_c)$ for each $e_i \in E$, and a Group Consensus Index GCI for the whole group. The feedback mechanism to update preferences must be applied if $ICI(P_i) > \mu$ for at least one $e_i \in E$, or $GCI > \lambda$, with μ and λ being the individual and group consensus threshold, respectively, with $\lambda \leqslant \mu$. In such a case, the assessments of discordant experts with the greatest differences among them are updated by assigning the corresponding value of the collective preference to them. This procedure is similar to the one previously shown in [80].

3.4. Q_4: no feedback mechanism and consensus measure based on distances between experts

Most automatic consensus models compute consensus degrees based on distances to the collective preference (see Section 3.3), but a small number of them carry out computations of similarities between pairs of experts to measure consensus. Some automatic and semi-automatic models based on computing distances

between experts (Fig. 13) are reviewed in this section, corresponding to the fourth quadrant of the taxonomy presented in this paper.

An adaptive consensus support system model inspired by the ideas of [27] was proposed by Chen et al. in [72]. Its main novelties with respect to the work of Mata et al. are: (i) preferences are given by intervals of linguistic 2-tuples, (ii) the system modifies preferences of experts by adjusting interval-valued assessments, and (iii) despite the underlying consensus model being automatic, the human expert can optionally decide to revise the changes applied to the preferences and accept them or not.

Zhang et al. extended in [73] the consistency-driven consensus model of Chiclana et al. [63], by introducing a linear optimization model to update preferences that ensures a minimum cost of modifying preferences, expressed as fuzzy preference relations. The main advantage of applying a linear optimization model is its low computational cost. Therefore, such a technique is utilized not only to conduct the CRP, but also to reach a high level of consistency for each individual preference relation.

In [16], Palomares et al. developed and presented a consensus support system based on a multi-agent architecture [82]. The main novelty of such a system is its capacity to automate the CRP completely, not only for the human moderator, but also for experts. To do this, experts provide their initial preferences (expressed as fuzzy preference relations) and delegate to autonomous software agents the revision of the advice received and the application of changes to preferences throughout the overall CRP. The underlying consensus model (see Fig. 14) follows some of the guidelines proposed in [27,60], such as: (i) the computation of pairwise similarities between experts by using the euclidean distance, (ii) the computation of consensus degrees at three levels, and (iii) although there is no real feedback for human experts, an agent-oriented feedback scheme consisting of identification and direction rules is implemented. Software agents are responsible for checking and applying direction rules on experts' preferences automatically. Moreover, two ontologies are defined and integrated in the model to facilitate communication and exchange of information amongst
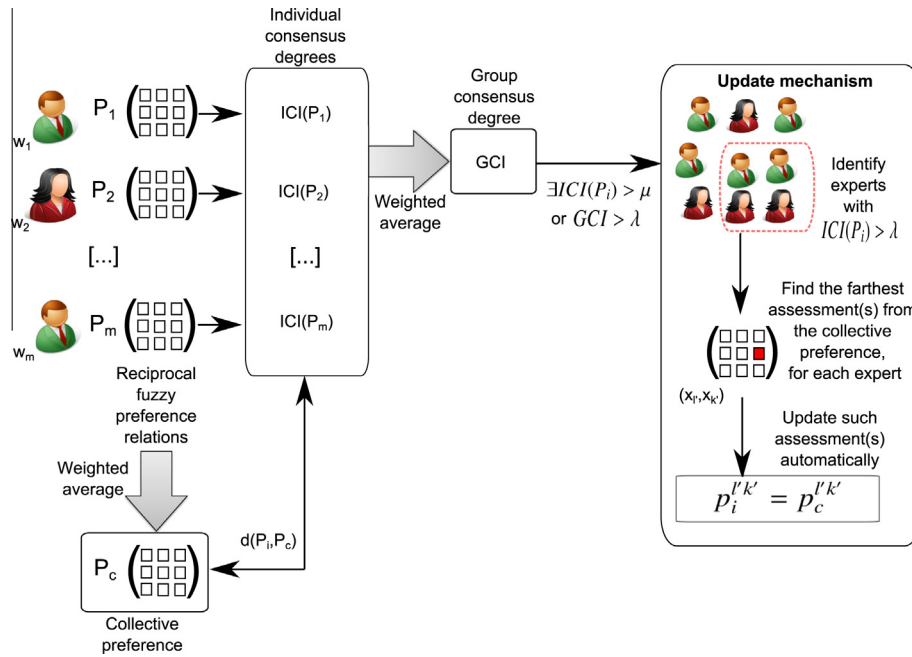
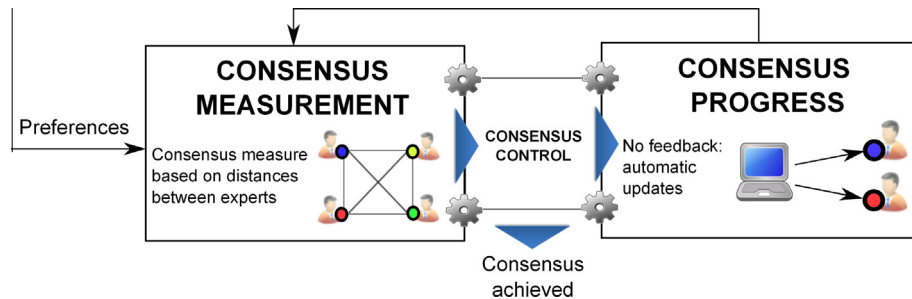**Fig. 12.** Consensus model of Xu et al. [21].

Fig. 13. General scheme of consensus models in Q_4.

agents, based on the ideas propounded by Kacprzyk and Zadrozny in [13]. Palomares et al. suggested the implementation and flexible use of different aggregation operators to measure consensus.

The system presented by Palomares et al. allows a full automation of human experts, regarding the process of supervising and modifying preferences. However, in [75], they argued that in some specific situations, it might be desirable that the human expert supervises the advice generated on an assessment p_i^{lk}, e.g. if such advice implies an important change to his/her preference. Based on this idea, they propose an agent-based semi-supervised approach that allows software agents to carry out most revisions of preferences by themselves, so that they only request human intervention when critical changes must be applied. Such an approach is based on the definition of several behavioral profiles that define how agents apply changes autonomously, as well as a rule-based mechanism to indicate the situations in which the human expert must revise his/her opinions. Its main advantage is the capacity of automating the CRP for human experts to a high degree, while preserving their sovereignty.

4. AFRYCA: A FRamework for the analYsis of Consensus Approaches

This section introduces a novel software framework called AFRYCA to simulate the resolution of GDM problems by using different consensus models proposed in the literature, many of which have been categorized and reviewed in the taxonomy previously presented. AFRYCA is mainly oriented towards a practical study of consensus models, for discovering the advantages and weaknesses of each model, analyzing the performance of a model under different settings, etc. The framework also aims at: (i) providing a better understanding of which models would be the most suitable to solve a specific type of GDM problem, and (ii) enabling comparisons between different consensus models, which could be useful to find out the main contributions of new proposals with respect to other existing works, for instance.

Firstly, we present the architecture and technologies of the framework (Section 4.1). A methodology for the use of the framework is then briefly described. Finally, we undertake a case study to show the performance of several consensus models implemented in the framework, for the resolution of several GDM problems (Section 5).

4.1. Architecture of AFRYCA

Here, the architecture of AFRYCA and the technologies that have been utilized in the analysis framework are presented.

AFRYCA has been developed under Java language, by means of the set of plugins *Rich Client Platform* (RCP), which enables the development of client desktop applications with rich functionality. One of the main advantages of RCP is its appropriateness for building component-based software applications based on high quality
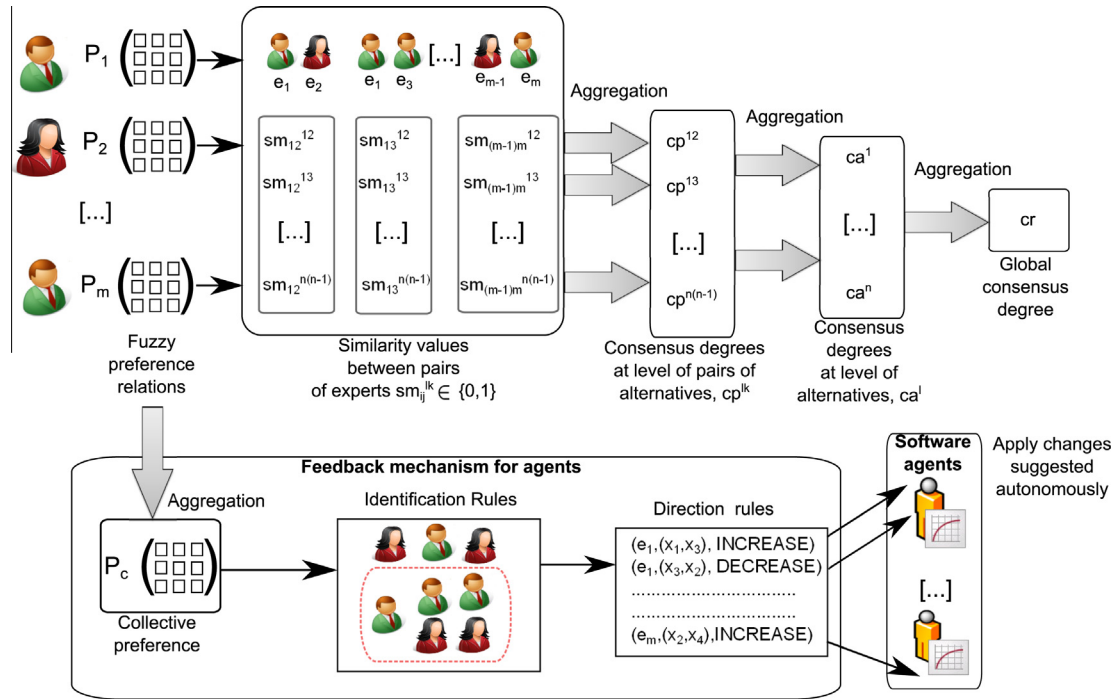
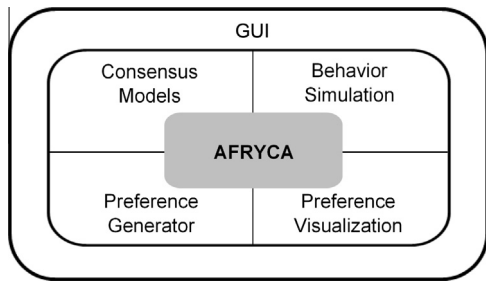**Fig. 14.** Agent-based consensus model of Palomares et al. [16].

Fig. 15. Architecture of AFRYCA.

components that are easy to maintain and extend, due to the high cohesion degree within each component and the low coupling between different components. Additionally, the software suite R[1] for statistical computing and graphics has been utilized to develop some components of the framework.

The framework is divided into five modules, as shown in Fig. 15. Such modules implement the functionalities and tools included in AFRYCA for the simulation and analysis of GDM problems based on consensus models, and they are described below:

- *Consensus Models*: Libraries that develop several existing consensus models. Each library corresponding to an existing consensus model is implemented in Java, and it includes the different phases (e.g. computation of consensus degrees, advice generation, etc.), operators (e.g. OWA, weighted mean, etc.) and parameters (e.g. consensus thresholds, linguistic quantifiers, etc.) necessary to apply such a model in practice. The flexible, loosely coupled architecture of AFRYCA facilitates the introduction of new libraries that implement additional consensus models easily. The current version of the framework incorporates the necessary libraries for using six consensus models based on the use of fuzzy preference relations:

- Three consensus models with feedback mechanism: Herrera-Viedma et al. [28] (see Fig. 6), Chiclana et al. [63] (see Fig. 8), and Kacprzyk et al. [68,13] (see Fig. 9).
- Three consensus models without feedback mechanism: Wu et al. [20] (see Fig. 11), Xu et al. [21] (see Fig. 12), and Palomares et al. [16] (see Fig. 14).

Remark 2. In AFRYCA, the current implementation of Herrera-Viedma et al.'s consensus model [28] omits the initial phase of unifying different preference structures, because the model deals with fuzzy preference relations only. Besides, in the model of Kacprzyk et al. in [68], the feedback mechanism based on linguistic summaries has been replaced by a feedback mechanism based on the criterion of "lack of arguments" suggested in [13].

- *Behavior Simulation*: This module has been designed to choose and simulate different patterns of behavior adopted by experts when accepting/ignoring feedback and modifying their assessments across the CRP. Such behavior patterns are utilized by the consensus models that have a feedback mechanism (see Sections 3.1 and 3.2). Two key aspects must be taken into account to define a behavioral pattern in AFRYCA. These two aspects are modeled by generating values belonging to different probability distributions, as follows:
  - The amount of recommendations on assessments that an expert e_i may accept or ignore. This feature can be modeled by means of a generator of discrete random values (e.g. 1 for *accept* or 0 for *ignore*) belonging to a probability distribution (e.g. binomial), whose parameter values (e.g. probability of success p in binomial distribution) can be fixed by the developer.
  - The degree of change that e_i may apply to the assessment p_i^{lk}, the modification of which he/she has accepted. This feature can be modeled with either a discrete or continuous probability distribution (e.g. Normal or Negative Binomial), so that values generated with R under this distribution represent the degree of change applied to the assessment.
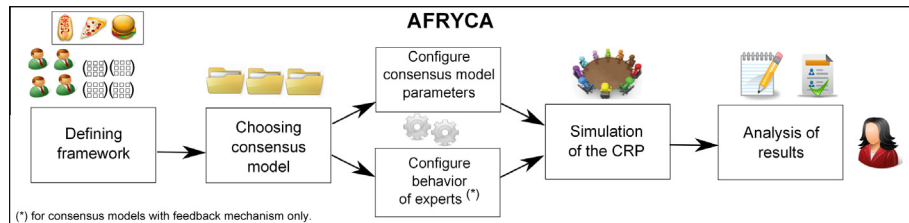
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[1] http://www.r-project.org.

Fig. 16. Methodology for the analysis of consensus models based on simulation of GDM problems.

A number of built-in *R* functions for the generation of random values under different probability distributions are utilized. R functions are invoked from Java code, by means of a third-party Java-R interface library. As occurred with consensus model libraries, this component can also be extended in the future. Moreover, such patterns can be used by different consensus models flexibly, in the sense that the user of AFRYCA may configure which behavioral pattern may be utilized with a specific consensus model at a given moment.

- *Preference Generator*: A Java implementation of the method proposed in [91] to construct consistent reciprocal fuzzy preference relations P_i from a set of $n-1$ values of assessments $p_i^{l(l+1)}, l \in \{1, \ldots, n-1\}$. Although such $n-1$ assessments are initialized randomly, the rest of the assessments are constructed taking into account the method mentioned above, thus ensuring consistency in preferences. This module allows the generation of data sets of experts' preferences. Each data set contains a specified number m of preference relations, as well as the formulation of a GDM problem, alternatives, etc. Such information is specified a priori, through the AFRYCA user interface. Data sets can be stored on a disk for future use.
- *Preference Visualization*: This module, inspired by the graphical monitoring tool of preferences presented in [92], provides a graphical 2-D representation of experts' preferences and the group preference, P_c, obtained after having conducted a CRP during the resolution of a GDM problem. Such a visualization is shown to the user of AFRYCA, together with the results of the GDM problem resolution. Some built-in R multi-dimensional scaling functions have been considered for the implementation of this module.
- *Graphical User Interface* (GUI): This allows users to interact with the rest of the modules in the framework. The GUI of AFRYCA has been implemented with the SWT (*Standard Widget Toolkit*) library, and it includes the necessary interfaces to: (i) choose the GDM problem and consensus model to utilize, (ii) configure the consensus model and select the behavioral pattern to simulate experts' behavior, (iii) visualize a summary of results after having applied the consensus model. It is also possible to generate a log file with more detailed results of the CRP conducted.

The architecture of AFRYCA offers several advantages, some of which are:

- Since it has been developed as a Java-based RCP, the framework can be used on any platform provided with a Java Virtual Machine, regardless of the operating system.
- The structure of AFRYCA, which is divided into separated modules, makes it possible to upgrade or extend some of its components (e.g. consensus model libraries and behavioral patterns, as mentioned above) without having to carry out changes that affect the whole framework.

A downloadable version of AFRYCA, as well as further details and documentation about the framework and its modules, can be found on the AFRYCA website.[2]

4.2. Methodology for using AFRYCA to simulate the resolution of GDM problems

Here, we describe the methodology for using AFRYCA to simulate the resolution of a GDM problem by using a consensus model implemented in the framework, and analyze different aspects of such a model, e.g. determining the strong points, weaknesses and types of GDM problems that can be solved with such a model, studying its performance with respect to other models, etc. The methodology is divided into the following steps, as depicted in Fig. 16:

1. *Defining Framework*: An instance of a GDM problem is chosen, to be solved by applying the consensus model previously chosen. To do so, the user can either select a data set file with an already existing GDM problem, or he/she can use the *Preference Generator* module to create a data set for a new GDM problem with m experts.
2. *Choosing consensus model*: A consensus model is chosen from amongst those included in the framework. The GUI of the framework provides a description and the main features of each model, as shown in Fig. 17.
3. *Configure parameters of the consensus model and behavior of experts*: Before proceeding to carry out the CRP, it is necessary to configure the values of parameters in the consensus model chosen (e.g. consensus thresholds, aggregation operators, etc.). For consensus models with a feedback mechanism, it is also necessary to specify the pattern of behavior adopted by experts when they receive recommendations and apply changes to their preferences (see *behavior simulation* module, Section 4.1).
4. *Simulation of the CRP*: Once the consensus model settings are fixed, the CRP is carried out.
5. *Analysis of results*: When consensus is achieved, an alternative selection process based on fuzzy non-dominance degrees of alternatives is applied [37], and the results of the GDM problem resolution are shown, in order to allow the user to analyze them. Results shown in the GUI include: (i) the initial consensus degree in the group and the final consensus degree achieved, (ii) the number of discussion rounds required, (iii) the ranking of alternatives and alternative/s chosen as the solution, and (iv) a visualization of experts' preferences and the group preference at the end of the CRP (see Fig. 18). AFRYCA also offers the possibility of storing a log file with more detailed results of the CRP performance.

5. Experimental study

In order to illustrate the purpose of AFRYCA, in this section we show an experimental study conducted to study the performance

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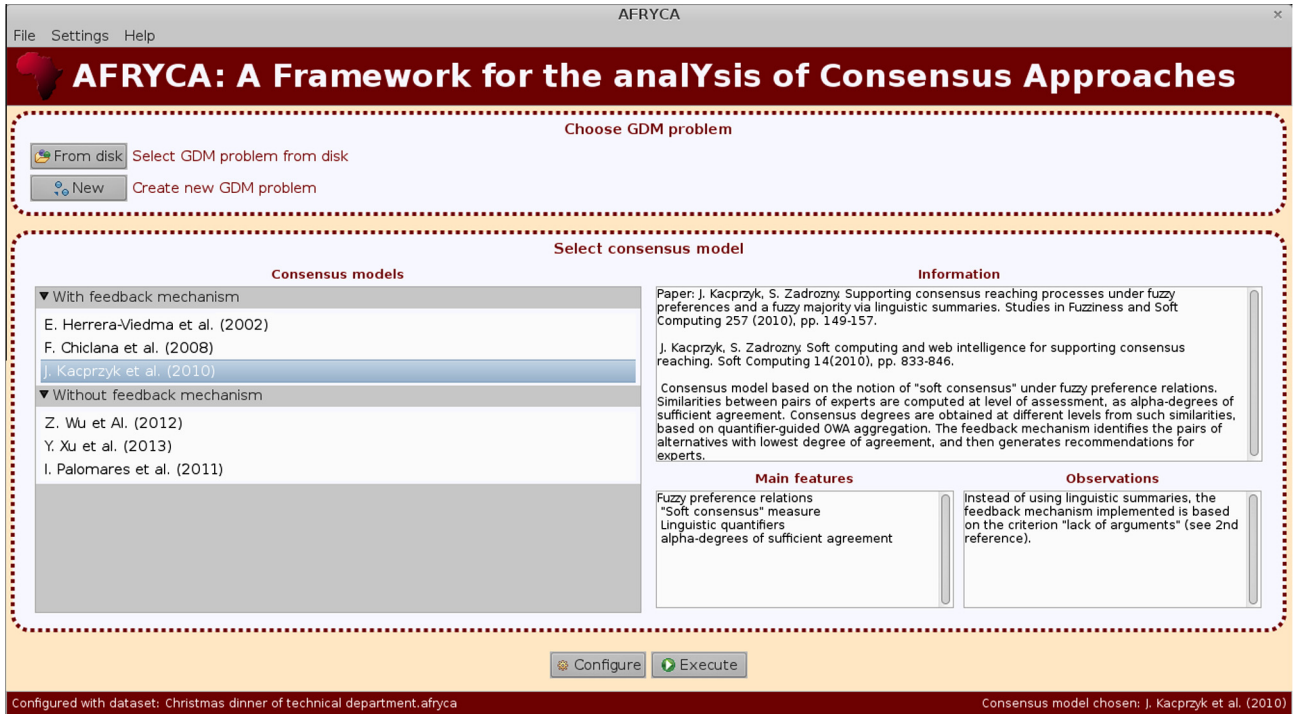
Fig. 17. Main interface of AFRYCA for the selection of a GDM problem and consensus model.
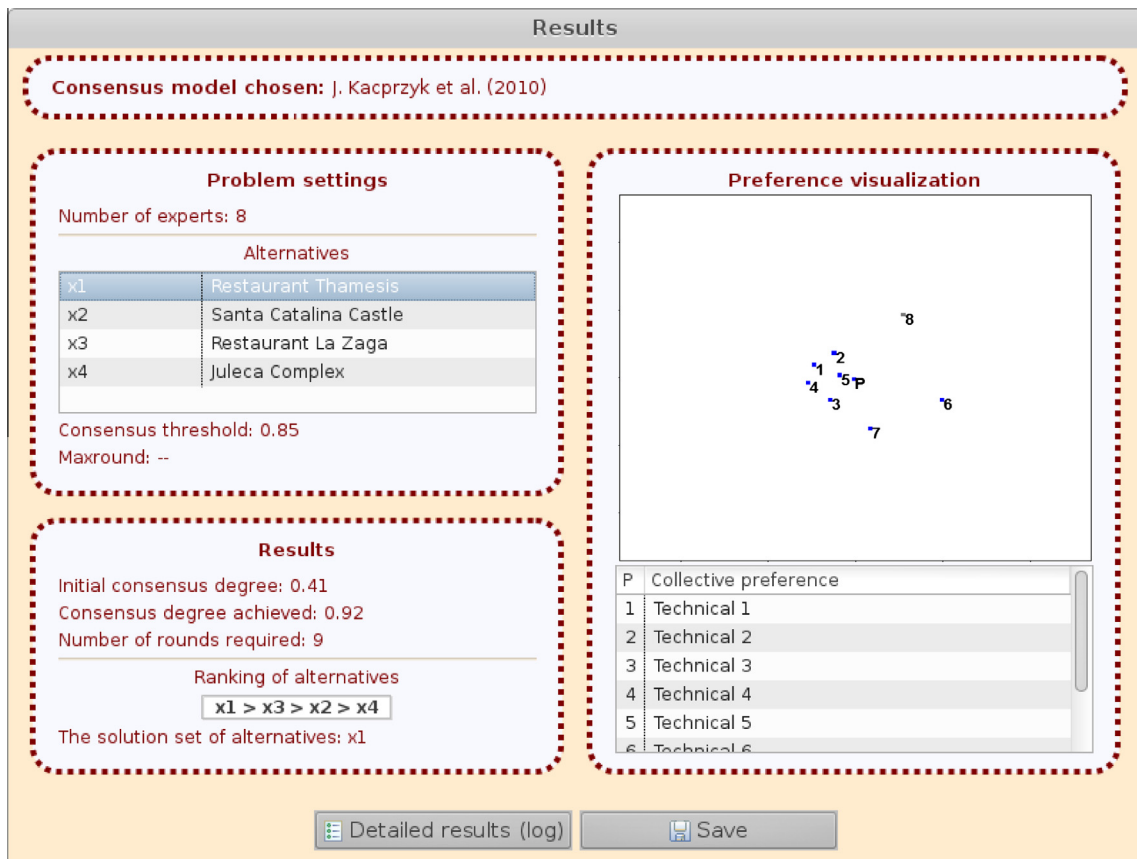
Fig. 18. Interface of results in AFRYCA.

of the consensus models integrated in the analysis framework [13,16,20,21,28,63,68], during the resolution of GDM problems with four different groups of experts.

Let us suppose a company composed of 32 employees, divided into four departments of equal size: Technical Department, $E_T = \{e_{T1}, \ldots, e_{T8}\}$, Human Resources Department,

$E_H = \{e_{H1}, \ldots, e_{H8}\}$, Marketing Department, $E_M = \{e_{M1}, \ldots, e_{M8}\}$ and Sales Department, $E_S = \{e_{S1}, \ldots, e_{S8}\}$. Each department plans to celebrate a Christmas dinner separately, hence each group must make a common decision about choosing a restaurant to celebrate their dinner, from amongst four possible alternatives for all of them: $X = \{x_1 :$ *Restaurant Thamesis*,
$x_2 :$ *St. Catalina Castle*, $x_3 :$ *Restaurant La Zaga*, $x_4 :$ *Juleca Complex*$\}$.

All experts express their preferences as fuzzy preference relations. These preferences are included in four data sets that will be used in this case study: one for each department. The minimum level of agreement required is $\mu = 0.85$ for all groups, and the maximum number of discussion rounds, *Maxround*, will not be taken into account in this case study, therefore all simulations will be carried out without the CRPs ending due to having exceeded the number of discussion rounds permitted.

The case study is divided into three parts: (i) simulation of consensus models with a feedback mechanism, (ii) simulation of consensus models without a feedback mechanism, and (iii) discussion of results. At each stage, the four GDM problems defined above are solved by means of three different consensus models. Then, the results obtained are analyzed and compared.

5.1. Consensus models with a feedback mechanism

The phases of the methodology shown in Section 4.2 to simulate CRPs and analyze the performance of consensus models are carried out for each GDM problem and consensus model separately:

(1) *Defining Framework.*

(2) *Choosing consensus model.*
(3) *Configure parameters of the consensus model and behavior of experts:* Table 2 summarizes the values chosen for parameters that need to be configured by the user of AFRYCA for each consensus model. Further information about such parameters, as well as the rules of the feedback mechanism and operations carried out during the different phases of the CRP, can be found in the reference associated to each model. Regarding the pattern utilized to simulate the behavior of experts in this case study, the degree of acceptance or rejection of recommendations to modify preferences is modeled by means of a Binomial Distribution, and the degree of change applied to accepted recommendations is modeled by means of Negative Binomial Distribution.
(4) *Simulation of the CRP.*
(5) *Analysis of Results:* The results of the performance of the CRP and the solution set of alternatives obtained with each consensus model, are summarized in Table 3. They will be discussed in Section 5.3.

5.2. Consensus models without a feedback mechanism

The previous methodology is applied again to solve the four GDM problems by means of each of the three consensus models without a feedback mechanism, with the only difference being that no experts' behavior needs to be configured for its simulation in the third phase.

(1) *Defining Framework.*

Table 2
Parameters of consensus models with a feedback mechanism.

	Herrera-Viedma et al. [28]	Chiclana et al. [63]	Kacprzyk et al. [68]
Consensus threshold	$\mu = 0.85$	$\mu = 0.85, \theta_1 = 0.75, \theta_2 = 0.8$	$\mu = 0.85$
Quantifier for aggregating information	F_{most}	–	$F_1 = F_2 = F_3 = F_{most}$
Quantifier for $QGNDD_l$	$F_{as\ many\ as\ possible}$	–	–
S_{OWA} OR-LIKE behavior	$\beta = 0.8$	–	–
S_{OWA} OR-LIKE behavior	$\beta = 0.8$	–	–
Recommendation rule in feedback mechanism	–	–	Lack of arguments [13]

Table 3
Results of the GDM problem resolution for consensus models with feedback mechanism.

	Herrera-Viedma et al. [28]	Chiclana et al. [63]	Kacprzyk et al. [68]
Technical Dept. (E_T)			
Initial consensus degree	0.79	0.77	0.41
Number of rounds	2	15	9
Final consensus degree	0.85	0.85	0.92
Ranking	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$
Alternative/s chosen	x_1	x_1	x_1
Human Res. Dept. (E_H)			
Initial consensus degree	0.76	0.69	0.1
Number of rounds	4	15	20
Final consensus degree	0.88	0.86	0.92
Ranking	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_2 \sim x_3 \succ x_4$	$x_1 \succ x_2 \succ x_3 \succ x_4$
Alternative/s chosen	x_1	x_1	x_1
Marketing Dept. (E_M)			
Initial consensus degree	0.78	0.63	0.11
Number of rounds	7	24	26
Final consensus degree	0.86	0.85	0.86
Ranking	$x_3 \succ x_1 \succ x_2 \succ x_4$	$x_1 \sim x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$
Alternative/s chosen	x_3	x_1, x_3	x_1
Sales Dept. (E_S)			
Initial consensus degree	0.71	0.61	0.09
Number of rounds	7	26	25
Final consensus degree	0.88	0.85	0.89
Ranking	$x_3 \succ x_1 \succ x_2 \succ x_4$	$x_1 \sim x_3 \succ x_2 \succ x_4$	$x_3 \succ x_2 \succ x_1 \succ x_4$
Alternative/s chosen	x_3	x_1, x_3	x_3

Table 4
Parameters of consensus models without a feedback mechanism.

	Wu et al. [20]	Xu et al. [21]	Palomares et al. [16]
Consensus threshold	$\mu = 0.15$	$\mu = 0.2, \lambda = 0.15$	$\mu = 0.85$
Normalized weights of experts	$w_i = 1/8, i = 1, \ldots, 8$	$w_i = 1/8, i = 1, \ldots, 8$	–
Updating coefficient	$\eta = 0.8$	–	–
Choice of aggregation operator	–	–	Arithmetic mean
Degree of change on assessments	–	–	0.05

(2) *Choosing consensus model.*

(3) *Configure parameters of the consensus model:* The values chosen for parameters that require configuration in AFRYCA for each consensus model are shown in Table 4. Notice that the consensus thresholds in [20,21] are distance-based thresholds, i.e. in this case consensus indices *below* these thresholds represent a satisfactory level of agreement, hence the values assigned to them are equal to $1 - \mu = 0.15$.

(4) *Simulation of the CRP.*

(5) *Analysis of Results:* Table 5 shows the results obtained from conducting the CRP with each consensus model and applying an alternatives selection process. In order to facilitate the comparison of consensus models, the consensus degrees shown in the table for the models of Wu et al. and Xu et al. are given by $1 - GCI$, because these models utilize group and individual distance-based consensus indices (denoted as *GCI* and *ICI* respectively, as shown in Section 3.3). The consensus degrees depicted in the table for the model of Wu et al. correspond to the *ICI* of the most distant expert in the group, i.e. $1 - \max_i ICI(P_i)$. The results are described in Section 5.3.

5.3. Discussion of the experimental study

Once the results of the experimental study have been set out, they are briefly discussed and analyzed, regarding their convergence towards agreement and the solution achieved.

From results of simulation with the consensus models with feedback mechanism (Section 5.1, Table 3), it can be observed that:

1. **Convergence**
   (a) The consensus model of Herrera-Viedma et al. presents a significantly higher convergence towards consensus for all the GDM problems, i.e. a lower number of consensus rounds are necessary to achieve the required level of agreement, $\mu = 0.85$.
   (b) The consensus model of Chiclana requires a large number of rounds to reach consensus, due to the values chosen for intermediate consensus thresholds θ_1 and θ_2, and the nature of its adaptive feedback mechanism, which generates a much lower amount of advice when the consensus degree exceeds θ_1.
   (c) Consensus degrees are much lower in the model of Kacprzyk et al., due to its similarity measure being based on α-degrees of sufficient agreement (see Eq. (2)), which is a rather strict measure.

2. **Solution**: The ranking of alternatives is very similar in the groups of experts belonging to the Technical and Human Resources Departments, with x_1 being the alternative chosen in both of them, regardless of the consensus model utilized. In the Marketing and Sales departments, either x_1 or x_3, or both of them, can be chosen as the solution to the GDM problem, depending on the model used.

Regarding the results of simulation with the consensus models without feedback mechanism (Section 5.2, Table 5), it can be observed that:

Table 5
Results of the GDM problem resolution for consensus models without feedback mechanism.

	Wu et al. [20]	Xu et al. [21]	Palomares et al. [16]
Technical Dept. (E_T)			
Initial consensus degree	0.7	0.84	0.77
Number of rounds	10	3	6
Final consensus degree	0.86	0.9	0.85
Ranking	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$
Alternative/s chosen	x_1	x_1	x_1
Human Res. Dept. (E_H)			
Initial consensus degree	0.67	0.79	0.69
Number of rounds	16	3	10
Final consensus degree	0.85	0.87	0.85
Ranking	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_2 \succ x_3 \succ x_4$
Alternative/s chosen	x_1	x_1	x_1
Marketing Dept. (E_M)			
Initial consensus degree	0.41	0.75	0.63
Number of rounds	19	4	14
Final consensus degree	0.86	0.89	0.86
Ranking	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_3 \succ x_1 \succ x_2 \succ x_4$	$x_3 \succ x_1 \succ x_2 \succ x_4$
Alternative/s chosen	x_1	x_3	x_3
Sales Dept. (E_S)			
Initial consensus degree	0.46	0.73	0.60
Number of rounds	20	4	12
Final consensus degree	0.85	0.87	0.86
Ranking	$x_3 \succ x_1 \succ x_2 \succ x_4$	$x_3 \sim x_2 \succ x_1 \succ x_4$	$x_3 \succ x_1 \succ x_2 \succ x_4$
Alternative/s chosen	x_3	x_3	x_3

1. **Convergence**
   (a) The convergence towards consensus is higher in the model of Xu et al., due to the fact that the identified assessments are directly updated with the value of the collective preference (see Fig. 12), therefore experts' preferences may experience significant changes in a single round.
   (b) The consensus model of Wu et al. applies small changes to preferences at each round (since $\eta = 0.8$ and the closer η is to 1, the smaller the changes applied [20]), hence its lower convergence.
   (c) The model of Palomares et al. also presents a lower convergence, because it has been applied with a low degree of autonomous change (increase/decrease) to assessments, 0.05.
2. **Solution**: x_1 is the best alternative at the Technical and Human Resources Departments, x_3 is the best alternative at the Sales Department, and either x_1 or x_3 could be the chosen alternative at the Marketing Department, depending on the consensus model.

We draw the following conclusions from the experimental case of study conducted:

- A similar solution is obtained at each group, regardless of the consensus model used for simulation: similar consensus degrees have been achieved, with slight differences in the alternative/s chosen as solution to the GDM problem.
- The main distinguishing element amongst the performances of consensus models, is the convergence that each one presents. Such a convergence is evaluated as the number of iterations or discussion rounds carried out before reaching a sufficient consensus degree. This could be an important factor for groups of experts, when they have to choose the most suitable consensus model in terms of usability.

6. Lessons learnt and future directions

The simulation of CRPs with AFRYCA provides multiples advantages and possibilities, some of which are:

- The framework makes it possible to simulate the resolution of a GDM problem under different consensus models, provided that they are suitable for dealing with such types of problems (e.g. consensus models for GDM problems with fuzzy preference relations). Thus, a decision maker, i.e. a person responsible for making the group decision, is able to study the performance and results obtained with each model.
- For a specific problem and consensus model, AFRYCA offers the possibility of investigating the different settings of such a model, based on the parameters or operators defined in it. Moreover, for those models with a feedback mechanism, the problem might be simulated under different patterns of expert behavior, in order to observe the effect of considering different types of behavior in the simulation.
- Although the decision group may prefer to conduct a real CRP, AFRYCA could provide them with a rough idea a priori about the performance of results that would be obtained, taking into account the initial preferences of experts and defining the appropriate problem settings that would reflect the real context of the problem.
- The experimental study presented has not focused on the use of different representational formats (e.g. linguistic preferences) to assess alternatives, but it is possible to implement and utilize any other existing types of preferences or representational formats in AFRYCA, for simulation purposes.

Six consensus models have been implemented in AFRYCA so far. Nevertheless, we note again that the architecture of the framework is designed to allow the inclusion of new consensus models (based on other types of preferences, information domains or even focused on MCGDM problems), as well as the further comparison between new models introduced and the existing ones.

Multiple proposals of consensus models have been presented in the specialized literature without showing a comparison with other existing models, hence their usefulness and main contributions are not justified properly. AFRYCA enables the implementation and analysis of these new proposals to find out their main contributions, with respect to the already existing ones.

Future work on extending the functionalities of AFRYCA, will mainly be oriented towards the definition of new metrics to measure the performance of a CRP. Such metrics would evaluate not only the discussion process itself, but also the quality of the collective solution achieved (in terms of its degree of acceptance by each member of the group, for instance), with the aim of facilitating a more comprehensive comparative study amongst different consensus models. This is currently one of the most important challenges in consensus: defining good performance measures would make it possible to evaluate the real usefulness of both new and existing proposals in the future.

7. Concluding remarks

Consensus has become a prominent research area in the field of group decision making. A large number of approaches to support consensus reaching have been proposed – and continue to be proposed – by a variety of authors.

In this paper, we have presented a taxonomy of existing consensus models for group decision making problems defined in a fuzzy context, which categorizes a number of consensus models based on their main characteristics, e.g. the type of information fusion techniques utilized to measure consensus in the group, or the procedures applied to increase the level of agreement throughout the discussion process. Besides characterizing a large number of existing consensus models, the taxonomy would also be useful to determine which could be considered for comparison with a new proposal, based on its characteristics and taking into account the taxonomy structure. Comparative studies are necessary to analyze the real capabilities of new proposals, instead of undertaking straightforward consensus exercises with them directly.

We have also presented a prototype of simulation-based analysis framework called AFRYCA, for the simulation of group decision making problems under consensus, by means of implementations of different existing consensus models in the literature. An experimental study has been shown to illustrate the usefulness of AFRYCA. To do this, six consensus models have been implemented and utilized in the study, based on the use of fuzzy preference relations to represent and manage preferences. As a result of the study conducted with AFRYCA, we suggest some future directions in the research topic of consensus: (i) the importance of comparing new proposals with existing ones, in order to show their contributions and (ii) the definition of new performance measures for consensus reaching processes, as a major challenge in the topic.

Finally, some recent approximations for consensus reaching consider different perspectives, e.g. agent-based consensus support systems [75], consensus models for large-scale group decision making problems [?,94], etc. These works could also be considered for their simulation in the framework.

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References

[1] J. Lu, G. Zhang, D. Ruan, F. Wu, Multi-Objective Group Decision Making, Imperial College Press., 2006.

[2] J. Kacprzyk, Group decision making with a fuzzy linguistic majority, Fuzzy Sets Syst. 18 (1986) 105–118.

[3] R. Bellman, L. Zadeh, Decision-making in a fuzzy environment, Manage. Sci. 17 (1970) 141–164.

[4] L. Zadeh, Fuzzy sets, Inform. Control 8 (1965) 338–353.

[5] L. Zadeh, Fuzzy logic equals computing with words, IEEE Trans. Fuzzy Syst. 4 (2) (1996) 103–111.

[6] F. Herrera, E. Herrera-Viedma, J. Verdegay, A sequential selection process in group decision making with linguistic assessments, Inform. Sci. 85 (1995) 223–239.

[7] M. Roubens, Fuzzy sets and decision analysis, Fuzzy Sets Syst. 90 (1997) 199–206.

[8] C. Butler, A. Rothstein, On Conflict and Consensus: A Handbook on Formal Consensus Decision Making, Takoma Park, 2006.

[9] S. Saint, J.R. Lawson, Rules for Reaching Consensus, A Modern Approach to Decision Making, Jossey-Bass, 1994.

[10] P. Eklund, A. Rusinowska, H. de Swart, A consensus model of political decision-making, Ann. Operat. Res. 158 (2008) 5–20.

[11] Z. Wu, J. Xu, Consensus reaching models of linguistic preference relations based on distance functions, Soft Comput. 16 (2012) 577–589.

[12] R. Parreiras, P. Ekel, J. Martini, R. Palhares, A flexible consensus scheme for multicriteria group decision making under linguistic assessments, Inform. Sci. 180 (2010) 1075–1089.

[13] J. Kacprzyk, S. Zadrozny, Soft computing and web intelligence for supporting consensus reaching, Soft Comput. 14 (2010) 833–846.

[14] C. Fu, M. Huhns, S. Yang, A consensus framework for multiple attribute group decision analysis in an evidential reasoning context, Inform. Fusion 17 (2014) 22–35 (Spec. Iss. Information fusion in consensus and decision making).

[15] Z. Gong, J. Forrest, Y. Yang, The optimal group consensus models for 2-tuple linguistic preference relations, Knowl.-Based Syst. 37 (2013) 427–437.

[16] I. Palomares, P. Sánchez, F. Quesada, F. Mata, L. Martínez, COMAS: a multi-agent system for performing consensus processes, in: Ajith Abraham et al. (Eds.), Advances in Intelligent and Soft Computing, vol. 91, Springer, 2011, pp. 125–132.

[17] E. Herrera-Viedma, F. Cabrerizo, J. Kacprzyk, W. Pedrycz, A review of soft consensus models in a fuzzy environment, Inform. Fusion 17 (2014) 4–13 (Spec. Iss. Information fusion in consensus and decision making).

[18] J. Kline, Orientation and group consensus, Cen. States Speech J. 23 (1972) 44–47.

[19] J. Kacprzyk, M. Fedrizzi, A "soft" measure of consensus in the setting of partial (fuzzy) preferences, Euro. J. Operat. Res. 34 (1988) 316–325.

[20] Z. Wu, J. Xu, A concise consensus support model for group decision making with reciprocal preference relations based on deviation measures, Fuzzy Sets Syst. 206 (2012) 58–73.

[21] Y. Xu, K. Li, H. Wang, Distance-based consensus models for fuzzy multiplicative preference relations, Inform. Sci. 253 (2013) 56–73.

[22] G. Bordogna, M. Fedrizzi, G. Pasi, A linguistic modeling of consensus in group decision making based on OWA operators, IEEE Trans. Syst., Man Cybernet., Part A: Syst. Hum. 27 (1) (1997) 126–133.

[23] F. Herrera, E. Herrera-Viedma, J. Verdegay, A rational consensus model in group decision making using linguistic assessments, Fuzzy Sets Syst. 88 (1) (1997) 31–49.

[24] F. Herrera, E. Herrera-Viedma, J. Verdegay, A model of consensus in group decision making under linguistic assessments, Fuzzy Sets Syst. 78 (1996) 73–87.

[25] F. Herrera, E. Herrera-Viedma, J. Verdegay, Linguistic measures based on fuzzy coincidence for reaching consensus in group decision making, Int. J. Approx. Reason. 16 (1997) 309–334.

[26] M. Brunelli, M. Fedrizzi, M. Fedrizzi, Fuzzy m-ary adjacency relations in social network analysis: optimization and consensus evaluation, Inform. Fusion 17 (2014) 36–45 (Spec. Iss. Information fusion in consensus and decision making).

[27] F. Mata, L. Martínez, E. Herrera-Viedma, An adaptive consensus support model for group decision-making problems in a multigranular fuzzy linguistic context, IEEE Trans. Fuzzy Syst. 17 (2009) 279–290.

[28] E. Herrera-Viedma, F. Herrera, F. Chiclana, A consensus model for multiperson decision making with different preference structures, IEEE Trans. Syst., Man Cybernet., Part A: Syst. Hum. 32 (2002) 394–402.

[29] R. Parreiras, P. Ekel, F. Bernardes Jr., A dynamic consensus scheme based on a nonreciprocal fuzzy preference relation modeling, Inform. Sci. 211 (2012) 1–17.

[30] C. Fu, S. Yang, The group consensus based evidential reasoning approach for multiple attributive group decision analysis, Euro. J. Operat. Res. 206 (2010) 601–608.

[31] A. Choudhury, R. Shankar, M. Tiwari, Consensus-based intelligent group decision-making model for the selection of advanced technology, Dec. Supp. Syst. 42 (2006) 1776–1799.

[32] N. Bryson, Group decision-making and the analytic hierarchy process. Exploring the consensus-relevant information content, Comp. Operat. Res. 23 (1996) 27–35.

[33] T. Tanino, Fuzzy preference orderings in group decision making, Fuzzy Sets Syst. 12 (1984) 117–131.

[34] S. Chen, J. Liu, H. Wang, J. Augusto, Ordering based decision making – a survey, Inform. Fusion 14 (2013) 521–531.

[35] F. Herrera, L. Martínez, P. Sánchez, Managing non-homogeneous information in group decision making, Euro. J. Operat. Res. 166 (2005) 115–132.

[36] D.-H. Peng, C.-Y. Gao, L.-L. Zhai, Multi-criteria group decision making with heterogeneous information based on ideal points concept, Int. J. Comput. Intell. Syst. 6 (2013) 616–625.

[37] S. Orlovsky, Decision-making with a fuzzy preference relation, Fuzzy Sets Syst. 1 (1978) 155–167.

[38] T. Saaty, Group decision making and the AHP, in: The Analytic Hierarchy Process: Application and Studies, Springer, Berlin, 1989, pp. 59–67.

[39] C. Fu, S. Yang, An evidential reasoning based consensus model for multiple attribute group decision analysis problems with interval-valued group consensus requirements, Euro. J. Operat. Res. 223 (2012) 167–176.

[40] F. Herrera, E. Herrera-Viedma, Linguistic decision analysis: steps for solving decision problems under linguistic information, Fuzzy Sets Syst. 115 (2000) 67–82.

[41] R. Rodríguez, L. Martínez, An analysis of symbolic linguistic computing models in decision making, Int. J. Gen. Syst. 42 (2013) 121–136.

[42] L. Martínez, J. Montero, Challenges for improving consensus reaching process in collective decisions, New Math. Nat. Comput. 3 (2007) 203–217.

[43] B. Spillman, J. Bezdek, R. Spillman, Development of an instrument for the dynamic measurement of consensus, Commun. Monog. 46 (1979) 1–12.

[44] D. Ben-Arieh, Z. Chen, Linguistic labels aggregation and consensus measure for autocratic decision-making using group recommendations, IEEE Trans. Syst., Man Cybernet., Part A: Syst. Hum. 36 (1) (2006) 558–568.

[45] J. Kacprzyk, M. Fedrizzi, H. Nurmi, Group decision making and consensus under linguistic preferences and fuzzy majority, Fuzzy Sets Syst. 49 (1992) 21–31.

[46] F. Herrera, E. Herrera-Viedma, J. Verdegay, Direct approach processes in group decision making using linguistic OWA operators, Fuzzy Sets Syst. 79 (1996) 175–190.

[47] D. Ben-Arieh, Z. Chen, Linguistic group decision-making: opinion aggregation and measures of consensus, Fuzzy Optim. Dec. Mak. 5 (2006) 371–386.

[48] L. Zadeh, A computational approach to fuzzy quantifiers in natural languages, Comput. Math. Appl. 9 (1983) 149–184.

[49] E. Szmidt, J. Kacprzyk, A consensus-reaching process under intuitionistic fuzzy preference relations, Int. J. Intell. Syst. 18 (2003) 837–852.

[50] R. Yager, On orderer weighted averaging aggregation operators in multi-criteria decision making, IEEE Trans. Syst., Man Cybernet. 18 (1988) 183–190.

[51] S. Khorshid, Soft consensus model based on coincidence between positive and negative ideal degrees of agreement under a group decision-making fuzzy environment, Exp. Syst. Appl. 37 (2010) 3977–3985.

[52] J. Chen, C. Chen, C. Wang, X. Jiang, Measuring soft consensus in uncertain linguistic group decision-making based on deviation and overlap degrees, Int. J. Innov. Manage., Inform. Product. 2 (2011) 25–33.

[53] R. Rodríguez, L. Martínez, F. Herrera, Hesitant fuzzy linguistic term sets for decision making, IEEE Trans. Fuzzy Syst. 20 (2012) 1109–1119.

[54] E. Herrera-Viedma, J. García-Lapresta, J. Kacprzyk, M. Fedrizzi, H. Nurmi, S. Zadrozny (Eds.), Consensual Processes, Studies in Fuzziness and Soft Computing, vol. 267, Springer, 2011.

[55] C. Carlsson, D. Ehrenberg, P. Eklund, M. Fedrizzi, P. Gustafsson, P. Lindholm, G. Merkuryeva, T. Riissanen, A. Ventre, Consensus in distributed soft environments, Euro. J. Operat. Res. 61 (1992) 165–185.

[56] G. Zhang, Y. Dong, Y. Xu, H. Li, Minimum-cost consensus models under aggregation operators, IEEE Trans. Syst., Man Cybernet. – Part A: Syst. Hum. 41 (2011) 1253–1261.

[57] Z. Wu, J. Xu, A consistency and consensus based decision support model for group decision making with multiplicative preference relations, Dec. Supp. Syst. 52 (2012) 757–767.

[58] N. Bryson, Supporting consensus formation in group support systems using the qualitative discriminant process, Ann. Operat. Res. 71 (1997) 75–91.

[59] P. Eklund, A. Rusinowska, H. de Swart, Consensus reaching in committees, Dec. Supp. 178 (2007) 185–193.

[60] E. Herrera-Viedma, L. Martínez, F. Mata, F. Chiclana, A consensus support system model for group decision making problems with multigranular linguistic preference relations, IEEE Trans. Fuzzy Syst. 13 (2005) 644–658.

[61] E. Herrera-Viedma, S. Alonso, F. Chiclana, F. Herrera, A consensus model for group decision making with incomplete fuzzy preference relations, IEEE Trans. Fuzzy Syst. 15 (2007) 863–877.

[62] Y. Dong, G. Zhang, W. Hong, Y. Xu, Consensus models for AHP group decision making under row geometric means prioritization method, Dec. Supp. Syst. 49 (2010) 281–289.

[63] F. Chiclana, F. Mata, L. Martínez, E. Herrera-Viedma, S. Alonso, Integration of a consistency control module within a consensus model, Int. J. Uncert., Fuzz. Knowl.-Based Syst. 16 (2008) 35–53.

[64] Y. Jiang, Z. Xu, X. Yu, Compatibility measures and consensus models for group decision making with intuitionistic multiplicative preference relations, Appl. Soft Comput. 13 (2013) 2075–2086.

[65] F. Cabrerizo, S. Alonso, E. Herrera-Viedma, A consensus model for group decision making problems with unbalanced fuzzy linguistic information, Int. J. Inform. Technol. Dec. Mak. 8 (2009) 109–131.

[66] I. Pérez, F. Cabrerizo, M. Cobo, S. Alonso, E. Herrera-Viedma, Consensual processes based on mobile technologies and dynamic information, Consensual processes, Stud. Fuzz. Soft Comput. 267 (2011) 317–337.

[67] S. Alonso, I. Pérez, F. Cabrerizo, E. Herrera-Viedma, A linguistic consensus model for web 2.0 communities, Appl. Soft Comput. 13 (2013) 149–157.

[68] J. Kacprzyk, S. Zadrozny, Supporting consensus reaching processes under fuzzy preferences and a fuzzy majority via linguistic summaries, in: S. Greco, et al. (Eds.), Preferences and Decision, Studies in Fuzziness and Soft Computing, vol. 257, 2010, pp. 261–279.

[69] J. Kacprzyk, S. Zadrozny, On a concept of a consensus reaching process support system based on the use of soft computing and Web techniques, in: Proceedings of the 8th International FLINS Conference, Madrid, Spain, 2008, pp. 859–864.

[70] C. Fu, S. Yang, An attribute weight based feedback model for multiple attributive group decision analysis problems with group consensus requirements in evidential reasoning context, Euro. J. Operat. Res. 212 (2011) 179–189.

[71] H. Lee, Optimal consensus of fuzzy opinions under group decision making environment, Fuzzy Sets Syst. 132 (2002) 303–315.

[72] S. Chen, L. Lee, S. Yang, T. Sheu, Adaptive consensus support model for group decision making systems, Expert Syst. Appl. 39 (2012) 12580–12588.

[73] G. Zhang, Y. Dong, Y. Xu, Linear optimization modeling of consistency issues in group decision making based on fuzzy preference relations, Expert Syst. Appl. 39 (2012) 2415–2420.

[74] S. Chen, L. Lee, Autocratic decision making using group recommendations based on the ILLOWA operator and likelihood-based comparison relations, IEEE Trans. Syst., Man Cybernet. – Part A: Syst. Hum. 42 (2012) 115–130.

[75] I. Palomares, L. Martínez, A semi-supervised multi-agent system model to support consensus reaching processes, IEEE Trans. Fuzzy Syst. (2013), http://dx.doi.org/10.1109/TFUZZ.2013.2272588. in press.

[76] M. Xia, Z. Xu, J. Chen, Algorithms for improving consistency or consensus of reciprocal [0,1]-valued preference relations, Fuzzy Sets Syst. 216 (2013) 108–133.

[77] Z. Xu, An automatic approach to reaching consensus in multiple attribute group decision making, Comp. Indust. Eng. 56 (2009) 1369–1374.

[78] Z. Xu, X. Cai, Group consensus algorithms based on preference relations, Inform. Sci. 181 (2011) 150–162.

[79] Y. Dong, Y. Xu, H. Li, B. Feng, The owa-based consensus operator under linguistic representation models using position indexes, Euro. J. Operat. Res. 203 (2010) 455–463.

[80] J. Xu, Z. Wu, A discrete consensus support model for multiple attribute group decision making, Knowl.-Based Syst. 24 (2011) 1196–1202.

[81] J. Xu, Z. Wu, A maximizing consensus approach for alternative selection based on uncertain linguistic preference relations, Comp. Indust. Eng. 64 (2013) 999–1008.

[82] M. Wooldridge, An introduction to Multiagent Systems, second ed., John Wiley and Sons, 2009.

[83] R. Yager, Families of OWA operators, Fuzzy Sets Syst. 59 (1993) 125–148.

[84] F. Chiclana, E. Herrera-Viedma, S. Alonso, F. Herrera, A note on the estimation of missing pairwise preference values: a uninorm consistency based method, Int. J. Uncert., Fuzz. Knowl.-Based Syst. 16 (2008) 19–32.

[85] K. Arrow, A. Sen, K. S (Eds.), Handbook of Social Choice and Welfare, Elsevier, 2002.

[86] F. Herrera, L. Martínez, A 2-tuple fuzzy linguistic representation model for computing with words, IEEE Trans. Fuzzy Syst. 8 (2000) 746–752.

[87] L. Martínez, F. Herrera, An overview of the 2-tuple linguistic model for computing with words in decision making: extensions, applications and challenges, Inform. Sci. 207 (2012) 1–18.

[88] R. Yager, A new approach to the summarization of data, Inform. Sci. 28 (1982) 69–86.

[89] G. Shafer, A Mathematical Theory of Evidence, Princeton University Press, Princeton, NJ, 1976.

[90] Z. Xu, A method based on linguistic aggregation operators for group decision making with linguistic preference relations, Inform. Sci. 166 (2004) 19–30.

[91] F. Chiclana, E. Herrera-Viedma, S. Alonso, F. Herrera, Cardinal consistency of reciprocal preference relations: a characterization of multiplicative transitivity, IEEE Trans. Fuzzy Syst. 17 (2009) 14–23.

[92] I. Palomares, L. Martínez, F. Herrera, MENTOR: a graphical monitoring tool of preferences evolution in large-scale group decision making, Knowl.-based Syst. 58 (2014) 67–74.

[94] I. Palomares, J. Liu, Y. Xu, L. Martínez, Modelling experts' attitudes in group decision making, Soft Comput. 16 (2012) 1755–1766.